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NUMERICAL ADJUSTMENT OF PROFILE LINE ON MATHEMATICALLY APPROXIMATED TERRAIN SURFACE

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Abstract

The paper presents calculation method for co-ordinate profiles on the surface, which is described by a shape function of the second degree. Algorithm of regressive function SPLINE has been used for adjustment of profile line. Function SPLINE determines, which analysed point belongs to interval of interpolation curve and solves determined set of linear equations by the aid of Gauss-Jordan's method. The suggested calculation provides sufficient precision for practical application.

1. Introduction

The given terrain surface can be covered by regular or irregular net of points with known spatial co-ordinates. It should be described function of the first or the second degree. The proposed function, which assures simple interpretations of its parameters and effectiveness of numerical calculation, has the following form (Piasek, 2001/1), (Piasek, 2001/3):

$$z = L_1 Z_1 + L_2 Z_2 + L_3 Z_3 + P_1 L_1 (L_2 + L_3) + P_2 L_2 (L_1 + L_3) + P_3 L_3 (L_1 + L_2) \qquad (1)$$

where:

Z_1, Z_2, Z_3 - point elevations in triangle apexes (constants),

P_1, P_2, P_3 - parameters of curvatures,

L_1, L_2, L_3 - shape functions within triangle.

Adopting this function makes it unnecessary to solve equation sets while we get the possibility to control the derivative at the triangle boundary by varying a single parameter. By comparing the directional derivatives of functions of elevation for two adjacent triangles we obtain parameter values ensuring smooth transition of surface curvatures (Piasek, 2001/1).

For adjustment of profile line we adopt function SPLINE. The regression function SPLINE has been worked out by Shoenberg I.J. (1946). At the beginning this functions were called segment functions. Now, most popular are functions SPLINE of the third degree. By the aid of SPLINE function, interpolation can be described as joining a polynomial function of the third degree in defined subintervals of interpolation interval. The joining of functions gives continuity of function SPLINE and the first and the second derivative of it as well. Thus SPLINE gives smooth curve in the given points.

Lawson C. L. and Hanson R. J. in ref. (1974) presents development of SPLINE on the base of experimental research and obtained results. In order to calculate co-ordinates of the profile, they use function supported on equal length intervals. If basic points are covered with interpolated points, then interpolated curve contains each interpolated point. The decreasing quantity of basic points causes greater smoothness of curve. In this case the distances between interpolation points will be increased from considered curve.

Of course, for that case, they use additional criteria of minimisation of square sum of distances of interpolation points. These distances are considered from the sought curve. Interpolation curve presents “n” basic points as linear combinations of “n+2” independent basic third degree functions SPLINE. Coefficients of linear combinations are calculated by the method of linear regression.

2. SPLINE algorithm

Parameters of function SPLINE:
N – quantity of interpolation points,
X – distance of interpolation points,
Y – height of the current point, which will be calculated function (1),
Subroutine SPLINE is described in [5] and presented in this paper with corrections. On the base of carried out tests, quantity ratio of interpolation points to the basic points, it has been assumed as 4.

```
      ! 5! 6! 7! 8 ! . .
      SUBROUTINE SPLINE (N,X,Y)
C     IMPLICIT DOUBLE PRECISION (A-H,O-Z)
      DIMENSION X(100),Y(100),Z(100),A(102,5),D(102,7),
     & B(100),C(100)
      M = N
      N = N / 4
      IF(N.LT.2) N = 2
      CALL S110 (M,X,Y)
      IF(N.GE.M-2) N = M-2
      B(1) = X(1)
      B(N) = X(M)
      H = (B(N)-B(1)) / FLOAT(N-1)
      DO 100  I = 2,N-1
100   B(I) = B(1) + H*FLOAT(I-1)
      CALL S120 (M,A,B,X,H)
      CALL S130 (N,M,A,D,Y,Z)
      CALL S140 (N,C,D,Z)
      CALL S200 (N,M,B,C,X,Y,H)
      RETURN
      END
C
C     function P(T)
C
      FUNCTION FNP(T)
C     IMPLICIT DOUBLE PRECISION (A-H,O-Z)
      FNP = 0.25*T**3
      RETURN
      END
C
C     function Q(T)
C
      FUNCTION FNQ(T)
C     IMPLICIT DOUBLE PRECISION (A-H,O-Z)
      FNQ = 1.-0.75*(1.+T)*(1.-T)*(1.-T)
      RETURN
      END
```

In subroutine SPLINE after substitution N=M, instruction N=N/4 is performed. Then quantity of basic points will be four times less. This quantity can not be less then 2. In order to save this digit, subroutine carries out the instruction IF(N.LT.2)N=2.

```
C
C        reading of data from tables "X" i "Y
C
         SUBROUTINE S100 (M,X,Y)
C        IMPLICIT DOUBLE PRECISION (A-H,O-Z)
         DIMENSION X(100), Y(100)
         DIMENSION X1(20), Y1(20)
         DATA X1, / 0.,1.,2.,3.,4.,5.,6.,7.,8.,9.,10.,
     &   11.,12.,13.,14.,15.,16.,17.,18.,19. /
         DATA Y1 / 0.5,1.,3.5,4.2,5.5,8.2,8.5,10.2,10.9,11.5,
     &   11.6,11.5,10.6,9.5,7.2,5.5,3.2,2.1,0.9,0.0 /
         M = 20
         DO 101  I = 1,M
         X(I) = X1(I)
101       Y(I) = Y1(I)
C        reading of data according to ascending of "X"
C        determination of maximal values of "X" i "Y"
         SUBROUTINE S110 (M,X,Y)
C        IMPLICIT DOUBLE PRECISION (A-H,O-Z)
         DIMENSION X(100), Y(100)
         X0 = X(1)
         Y0 = Y(1)
         XM = X0
         YM = Y0
         DO 101  I = 2,M
         IF(X0.GT.X(I)) X0 = X(I)
         IF(XM.LT.X(I)) XM = X(I)
         IF(Y0.GT.Y(I)) Y0 = Y(I)
         IF(YM.LT.Y(I)) YM = Y(I)
101      CONTINUE
         ID = IDINT (DLOG(FLOAT(M)) / DLOG(2.))
         ID = 2**ID - 1
100      CONTINUE
         DO 103  I = 1,M-ID
         DO 102  J = I,1,-ID
         IF(X(J).LE.X(J+ID)) GO TO 103
         HY = Y(J)
         Y(J) = Y(J+ID)
         Y(J+ID) = HY
         HX = X(J)
         X(J) = X(J+ID)
102       X(J+ID) = HX
103      CONTINUE
         ID = ID / 2
         IF(ID.GT.0) GO TO 100
         RETURN
         END
```

The subroutine S110 determines and sorts of value X;Y. Next in table “B”, co-ordinates of basic points are calculated.

```
C          determination of elements of matrices  "A"
C
           SUBROUTINE S120 (M,A,B,X,H)
C          IMPLICIT DOUBLE PRECISION (A-H,O-Z)
           DIMENSION A(102,5), B(100), X(100)
           IB = 1
           IX = 1
101        CONTINUE
           IF(X(IX).GT.B(IB+1)) GO TO 100
           T = (X(IX) - B(IB)) / H
           A(IX,1) = FNP(1.-T)
           A(IX,2) = FNQ(1.-T)
           A(IX,3) = FNQ(T)
           A(IX,4) = FNP(T)
           A(IX,5) = FLOAT(IB)
           IF(IX.EQ.M)  GO TO 102
           IX = IX + 1
           GO TO 101
100         IB = IB + 1
           GO TO 101
102        CONTINUE
           RETURN
           END
```

The subroutine S120 creates matrix “A” for successive points. Distances between these points and origin of profile line are located in table “X”. Subroutine determines, which analysed point belongs to interval of interpolation curve “IB”. In interval “IB”, which assumes values from 0 to 1, local co-ordinate “T” of analysed point will be located. Calculation of coefficients of matrix “A” performs functions: FNP(T) and FNQ(T), which are defined in following form:
FNP=0.25*T**3   ;   FNQ=1.-0.75*(1.+T)*(1.-T)*(1.-T).
Rectangular matrices “A” has sizes A(252,5).

```
C
C          determination of elements of strip matrices  "D" i "Z"
C
           SUBROUTINE S130 (N,M,A,D,Y,Z)
C          IMPLICIT DOUBLE PRECISION (A-H,O-Z)
           DIMENSION A(102,5), D(102,5), Z(100), Y(100)
           DO 100  I = 1,N+2
           DO 101  J=1,7
101        D(I,J) = 0
100        Z(I) = 0.
           DO 102  K = 1,M
           DO 102  I = 1,4
           DO 103  J = 1,I
           I0 = I + IDINT (A(K,5)) - 1
           J0 = J + IDINT (A(K,5)) - 1
103        D(I0,J0 - I0 + 4) = D(I0,J0 - I0 + 4) + A(K,I)*A(K,J)
102        Z(I0) = Z(I0) + A(K,I)*Y(K)
           DO 104  I = 1, N+2
           DO 104  J = 1,3
           IF(D(I,J).NE.0.)  D(J + I - 4, 8 - J) = D(I,J)
104        CONTINUE
           RETURN
           END
```

Subroutine S130 calculates:

- strip matrices “D” as product of transposed matrices to “A”; D= A^T *A,
- vector “Z” as product of transposed matrices to “A” by height of vector Y; Z= A^T *Y.

Width of strip of considered matrices is equal to 7; D=7. By this method will be created set of linear equations with unknown coefficients of linear combinations of basic function SPLINE.

```
C
C          solution of equations set  "D*C = Z"
C
           SUBROUTINE S140 (N,C,D,Z)
C          IMPLICIT DOUBLE PRECISION (A-H,O-Z)
           DIMENSION D(102,7), Z(100), C(100)
           DO 100  I = 1,N+2
           D4 = D(I,4)
           DO 101  J = 1,7
101         D(I,J) = D(I,J) / D4
           Z(I) = Z(I) / D4
           J = I + 3
           IF(J.GT.N+2) J = N + 2
           IF(I.EQ.N+2)  GO TO 100
           DO 103  K = I + 1, J
           XL3 = D(K,4 - (K - I))
           DO 104  L = 1,4
           L1 = L + 3
           L2 = L + 3 - (K - I)
104         D(K,L2) = D(K,L2) - XL3*D(I,L1)
103         Z(K) = Z(K) - XL3*Z(I)
100         CONTINUE
           DO 105  I = 1,N+5
105         C(I) = 0.
           DO 106  I = N + 2, 1, - 1
106         C(I) = Z(I)-D(I,5)*C(I+1)-D(I,6)*C(I+2)-D(I,7)*C(I+3)
           RETURN
           END
```

Subroutine S140 solves determined set of linear equations by the aid of Gauss-Jordan’s method. It is a popular method for solving strip matrices. As before, the set of linear equations is stored in table “C”.

```
C
C          determination of function  "SPLINE"
C
           SUBROUTINE S200 (N,M,B,C,X,Y,H)
C          IMPLICIT DOUBLE PRECISION (A-H,O-Z)
           DIMENSION B(100), C(100), X(100), Y(100)
           NSTEP = 2
           DO 100  I = 1,M
           DO 100  K = 1,NSTEP
           IF (I . EQ . M . AND . K . GT . 1)  GO TO 100
           XX = X(I)+(K - 1)*(X(I + 1) - X(I)) / NSTEP
C          XX = X(I)
           CALL S210 (N,J,XX,B)
           T = (XX - B(J)) / H
           YY=C(J)*FNP(1.-T)+C(J+1)*FNQ(1.-T)+C(J+2)*FNQ(T)+
         &  C(J+3)*FNP(T)
           WRITE (6,101)  I, XX, Y(I), YY
100        CONTINUE
```

```
101       FORMAT (1X,I3,F15.1,F16.1,G25.7)
          RETURN
          END
```
Subroutine S200 calculates height of given point by means of equation:

YY=C(J)*FNP(1.-T)+C(J+1)*FNQ(1.-T)+C(J+2)*FNQ(T)+C(J+3)*FNP(T)

```
C
C         determination of current distances to function "SPLINE"
C
          SUBROUTINE S210 (N,J,XX,B)
C         IMPLICIT DOUBLE PRECISION (A-H,O-Z)
          DIMENSION B(100)
          J = 0
          IF (XX . LE . B(1)) THEN
          J = 1
          RETURN
          ENDIF
          IF(XX . GE . B(N) THEN
          J = N - 1
          RETURN
          ENDIF
100        J = J + 1
          IF (XX . GT . B(J))  GO TO 100
          J = J - 1
          RETURN
          END
```
File with results contains:

I – number of point,
XX – current co-ordinate of point,
Y(I) – value of height of interpolation point,
YY – calculated value

3. Testing of function SPLINE and conclusions

The testing based on a hemisphere cap (R=5), which was approximated function of shape (1). The first stage of the test has been performed in work (Piasek, 2001/3). Obtained calculation accuracy of profile points was equal to 0.175 and it gives correct profile line.

The second stage of calculation involves improvement of accuracy of profile line for engineering task. On diameter of hemisphere cap were performed calculation of points by the aid of analysed subroutine SPLINE. Assumed extreme quantities of basic points are equal to 20 and 100. Similarly as in (Piasek, 2001/3), as a criterion of calculation accuracy of profile points heights, we assume average value of absolute error of calculated heights in given points related to analytically calculated heights.

Table1. Heights of points on the hemisphere cap calculated analytically (REAL) and subroutine SPLINE

Polar co-ordinates	REAL	SPLINE	SPLINE
0.0	0.0	-0.21	-0.01
0.5	2.18	1.44	0.95
1.0	3.0	2.64	2.86
1.5	3.57	3.48	3.68
2.0	4.0	4.04	4.02
2.5	4.33	4.42	4.45
3.0	4.58	4.69	4.53
3.5	4.77	4.87	4.78
4.0	4.90	4.96	4.92
4.5	4.97	5.00	5.01
5.0	5.00	5.00	5.04
		Base points	Quantity
		20	10
		Accuracy of	Profile points
		0.166	0.163

From table 1, it is shown that using subroutine SPLINE, we obtain calculation accuracy equal to 0.166 for 20 basic points and 0.163 for 100 basic points. In the first case, improvement of accuracy is about 5% and in the second - 6.9%. We can assume that 20 basic points is sufficient for correction of the profile line.

After testing of SPLINE algorithm, the conclusions are as follow:

- subroutine has simple interpretation for practical application,
- quantity of basic pints can be arbitrary changed,
- improvement of accuracy of the profile line is useful in practice.

References:

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