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ESTIMATION OF DIVERGENCES IN PRECAST CONSTRUCTIONS USING GEODETIC CONTROL NETWORKS

George D. Georgopoulos & Elisavet C. Telioni

National Technical University of Athens, Rural & Surveying Engineering School, Department of Topography, Laboratory of General Geodesy, 9,Heroon Polytechniou, 157 80, Athens, Greece

Abstract

The divergences of the geometric features of the concrete elements are usually detected after the construction's erection. These divergences are of prominent importance in precast concrete construction, since they might prove responsible for the deterioration of its bearing ability. Using geodetic methodology, they can be reliably estimated and their values compared against the permitted tolerances, set from the precast construction rules. For this purpose, a geodetic control network is established and its elements are measured using instruments of high precision. After the adjustment of the observations, the vectors of divergences are estimated and their statistical significance is tested. The special requirements in the design and optimization of such a network are analyzed in this paper, and a case study dealing with the detection of the divergences of a two floor precast construction is presented. The finally estimated divergences were all statistically significant; their values ranged up to 52 *mm* and exceeded the permitted tolerances. Since the geodetic control network can also be used for the monitoring of the kinematic behavior of the construction, the possibility of using GPS measurements for this purpose is examined through the results of an experiment.

1. Introduction

During the construction of a concrete building, differences (named **divergences**) between the nominal values of its geometric features determined in the general construction plans, and those finally resulting after the building's erection are inevitable. In case that these divergences exceed the permitted **tolerances,** set in construction rules, deterioration of the building's bearing ability is possible. Especially in precast concrete constructions, divergences are of prominent importance since, if deformations occur after the building's erection, the redistribution of the strain forces is not possible (Andrianis, Petsas, 1983).

Divergences in precast construction may happen during the fabrication, the storage and the transportation of the concrete elements in the place of the building's construction. These divergences mainly concern the elements' dimensions and quality and can be minimized if special precautions are taken. The most significant and important divergences, from the point of view of the bearing ability of the building, are those occurring during the consecutive stages of the precast assembling (i.e. the building's erection). They consist of divergences due to incorrect positioning of the foundation-precast elements, declinations from vertical position and displacements of the precast elements.

The final total divergences in precast construction can be reliably estimated using geodetic methodology. For this purpose, a geodetic control network is established, its elements are measured using instruments of high precision, and the coordinates of the network's points are estimated. Finally, the divergences in the positioning of the precast elements, as well as those of the geometric features of the construction are determined and their significance is tested by appropriate statistical tests (Georgopoulos 2000).

2. Use of geodetic methodology for the estimation of reliable divergences in precast contsruction

The geodetic control network established for the detection of divergences in precast construction consists of the **reference** and the **control points**.

The answer to the **zero order design** problem of such a network is not necessary. Since the horizontal divergence or displacement vectors of the bearing frame of the construction are determined by the comparison of the coordinates of the network's points, located on the same precast element, they are relative and not absolute. Therefore the optimum reference system is a local geodetic coordinate system. The network's geodetic datum is defined by the minimum constraints i.e. fixing the coordinates of one of the network's points and the azimuth of the same "fixed" point to another point of the network, making the assumption that the coordinates system is centered to the "fixed" point. Special care must be taken for the selection of the fixed point. It should be located to the most stable area of the surroundings of the construction and its stability must be checked from time to time.

The **reference points**, located in the vicinity of the construction, are established during the stage of the building's setting out. Their number and position depends on the size and the geometric characteristics of the building and on the stability of the surrounding area. Since the reference points can be used from the beginning (setting out) till the end of the construction works, their monumentation should be permanent.

The **control points** are situated on the construction. Their location depends on the kind of the precast element on which they are established. For instance, in case of linear elements, such as columns, where the most significant divergences are the columns' declinations from vertical position, two or even three points are established on the symmetry axis of at least one side of each column. It has been proved in practice, that it would be very useful, if the marking of the control points is done at the stage of the elements' fabrication in the factory.

Therefore the answer to the **first order design** problem is rather imposed by the special geometric characteristics of the construction under consideration, than the optimum location of the network's points.

The **second order design** problem of such a network concerns primarily the accuracy of the observations, i.e. the selection of the instruments to be used. According to the predefined accuracy of the points' coordinates (usually of a few *mm*), the accuracy of the observations is determined and the instruments meeting these demands are chosen.

The design of the geodetic control network is optimum if it satisfies the criteria adopted to define the quality of the network. These criteria are the **requirements in precision and reliability** of the network (Agatza – Balodimou, 1999).

Since the geodetic control network is established for the estimation of the divergences of a precast construction, it is also important to determine the minimum horizontal divergence vector that can be estimated by the network. That means that if the postulated divergences occur, they can be detected with a specified confidence level. This is called the **sensitivity** of the network (Krakiwsky, 1991, Kuang 1991). The sensitivity of the geodetic control network, concerning the horizontal divergence vector between *i, j* control points, belonging to the same precast element, is determined through the following equation (Georgopoulos 2000):

$$min\,\delta r_{i,j} = z_{0.99} \cdot \sigma_{\delta r\,max} \tag{2.1}$$

where:

$\delta r_{i,j}$: the horizontal divergence of the precast element, as determined through the coordinates' differences between $i,\ j$ control points ($\left|\delta r_{i,j}\right| = \sqrt{\delta x_{i,j}^2 + \delta y_{i,j}^2}$), and

$\sigma_{\delta r\,max}$: the major semi – axis of the error ellipse of the divergence vector, determined through the a priori covariance submatrix $\boldsymbol{V}_{\delta r_{i,}}$.

The adjustment of the observations leads to the estimation of the points' coordinates and their standard errors. After the adjustment, the well known global test on the a posteriori $\hat{\sigma}_0$ and

Baarda's data snooping (on the observations) are applied in order to check the precision and reliability of the network (Krakiwsky 1991, Kuang 1991).
If the null hypothesis H_0 (the mathematic and stochastic part of the model are both correct) is accepted, the divergence vectors of the construction are determined, and their statistical significance is tested for a specified confidence level p% (usually p = 95%).
The horizontal divergence $D_{i,j}$ of each precast element is determined through the coordinates' differences of the $i, \ j$ control points established for this reason on the element:

$$D_{i,j} = \sqrt{\left(X_j - X_i\right)^2 + \left(Y_j - Y_i\right)^2} \tag{2.2}$$

The above computed divergence is statistically significant if the following inequality holds (Krakiwsky 1991, Kuang 1991):

$$D_{i,j} > \sigma_{D_{i,j}} \cdot z_{0.95} \tag{2.3}$$

where:
$\sigma_{D_{i,j}}$: the standard deviation of the divergence computed through the errors' law of propagation,
$z_{0.95}$: the corresponding value of the normal distribution for confidence level 95%.

3. A case study

In order to estimate the divergences of a two floor industrial precast building, having dimensions $20m \times 35.50m$, the above described geodetic methodology was applied. The bearing frame of the construction consists of 15 precast concrete columns, while its walls are precast elements too.
During the stage of the network's design its geometry was determined. The network consists of **42** points: **8 reference points** (K_1, …, K_8) and **34 control points,** two on each precast column. The control points are located on the symmetry axis of the column's external side, except of three of the four columns at the corners of the construction**.** At these columns four control points were established, two on each external side.
The observation accuracies determined at the stage of the network's design were $\pm 3mm$ and 10^{cc} for the length and angle measurements respectively. The trace of the a priori covariance matrix (tr $V_{\hat{x}}$) of the coordinates, the mean standard deviation (σ) of the coordinates estimation and the minimum detectable horizontal divergence ($min D$) for confidence level 99% were also determined at this stage.
The instrument used for the observations was the Total Station TC1600 Wild, having an accuracy of 0.3mgon in angle measurements and $\pm(3mm \pm 3ppm)$ in length measurements. 110 length and 108 angular measurements formed the observational scheme. The shape of the network is depicted in Fig.1.
The adjustment of the observations was performed with the minimum external constraints: the coordinates of point K_1 together with the azimuth of side K_1K_5 were kept fixed (X_{K_1} (= Y_{K_1} + 100.000m, $az_{K_1-K_5} = 100^g$). From the adjustment of the observations, the estimates of the coordinates of the network's points were determined, together with their covariance matrix $\hat{V}_{\hat{x}}$. The null hypothesis was accepted from the global test of the network, while no observation was rejected when Baarda's data snooping was applied.
The heights of the control points were determined by trigonometric height leveling from the reference points, whose heights were determined by double geodetic leveling. In order to detect the possible vertical diferential displacements of the upper parts of the columns, their heights were also determined by double geodetic leveling. For the determination of heights of all the mentioned above points, the height of point K_1 was kept fixed (H_{K_1} = + 100.000m).

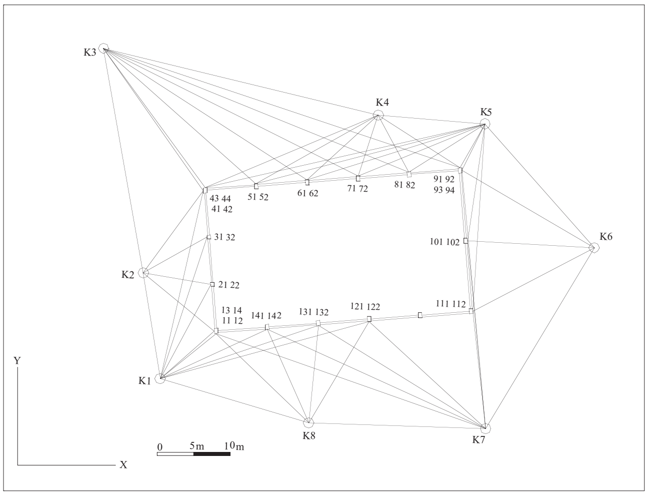

Fig. 1. The established geodetic control network for the detection of the divergences of a two-floor precast concrete building.

For each column the coordinates of the upper control point were compared to those of the lower one, and the corresponding vector of horizontal divergence was calculated from equation (2.2). Its statistical significance was tested for significance level 95%.

All the horizontal divergence vectors were proved to be statistically significant. They range from $11mm$ (or $0.001 rad$) up to $52 mm$ (or $0.008 rad$). It must be pointed out that *the permitted tolerance of divergences,* according to precast construction rules (Technical Chamber of Greece, 1991), is $\underline{D_D \leq \pm 20mm}$ or $\underline{D_a \leq 0.003 rad}$**,** therefore the estimated divergences are significantly greater than the permitted ones**.** The determined horizontal divergence vectors in (mm) and (rad) together with their azimuths are depicted in Table 1.

Column	Control Points	Linear divergence (mm)	Distance of control points (m)	Angular divergence (rad)	Azimuth (g)
1	11 – 12	20	6.589	0.003	87.43
	13 – 14	**22**	6.770	0.003	79.52
2	21 – 22	**24**	7.272	0.003	133.05
3	31 – 32	**24**	7.908	0.003	89.49
4	41 – 42	**41**	8.673	**0.005**	4.65
	43 – 44	**42**	8.747	**0.005**	13.76
5	51 – 52	**35**	10.514	0.003	140.97
6	61 – 62	**23**	10.550	0.002	144.23
7	71 – 72	15	10.645	0.001	140.97
8	81 – 82	11	10.328	0.001	194.23
9	91 – 92	**52**	10.554	**0.005**	138.78
	93 – 94	**49**	9.367	**0.005**	135.29
10	101 – 102	**23**	8.430	0.003	344.23
11	111 – 112	**27**	7.691	**0.004**	9.36
12	121 – 122	**42**	4.990	**0.008**	353.23
13	131 – 132	**25**	6.033	**0.004**	371.60
14	141 - 142	14	6.439	0.002	333.62

Table 1. Horizontal divergence vectors of the construction's columns.
(divergences that exceed the permitted tolerances are given with bold characters).

Statistically significant vertical divergences, concerning the co-horizontality of the columns' upper parts, were also determined, the maximum one being of 33 *mm*.
In Figure 2. the horizontal divergence vectors of the construction's columns together with the vertical ones are depicted.
Divergences from the nominal values, as given from the construction plans, and concerning various geometric features of the construction, such as distances between the columns' symmetry axis, the size of the building's parts etc., were also determined and found greater than the preset tolerances.

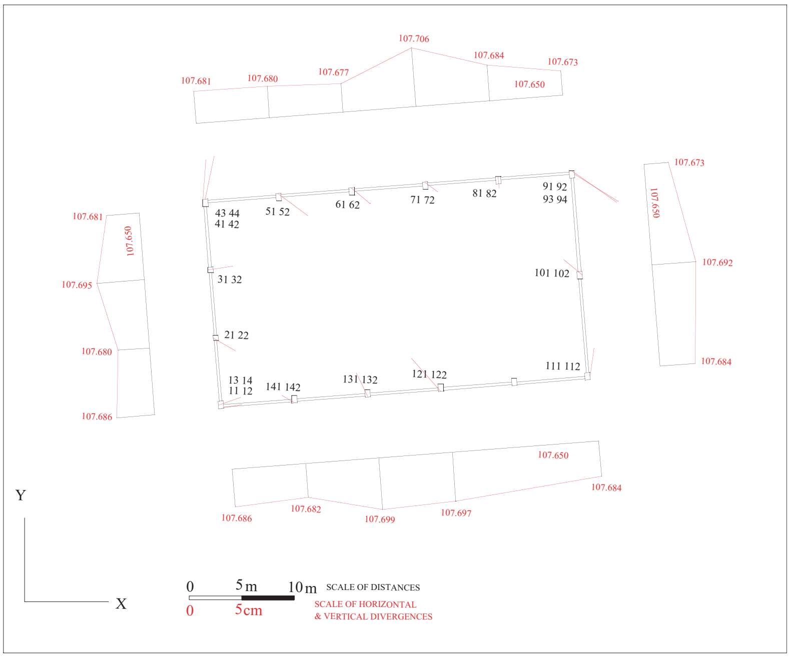

Fig.2 Horizontal and vertical divergences of the columns of the two floor precast construction.

4. Use of GPS measurements for the determination of the horizontal movement of the building's roof slab

The use of GPS technology for the detection of divergences in precast construction is limited due to the position of the control points. It could however be used for the determination of the sides of the reference control network, and thus ameliorate the overall accuracy of the network. Moreover, since the geodetic control network established for the detection of the vectors of divergences in a precast construction, can also be used for the monitoring of its kinematic behavior, if deformations occur after its erection, it is interesting to investigate the possibility of using GPS measurements for the roof slab's horizontal movements.
For this purpose an experiment was carried out at the roof slab of the aforementioned building. Three control points A_1, A_2, A_3 were established on this slab, forming a triangle with sides of the range of 12*m*. Thus a new control network was established, consisting of the 8 reference points and the 3 new control ones. Its sides were determined by GPS measurements, and the coordinates of the points were estimated (Fig 3). Three more control points B_1, B_2, B_3 were, afterwards, established in such a way that the distances A_1B_1, A_2B_2, A_3B_3 are of some cm and the sides of the triangle $B_1B_2B_3$ have equal lengths to those of the triangle $A_1A_2A_3$. Thus a "pseudo" shift and rotation of the roof's slab occurred (Fig.3).
The coordinates of the points B_1, B_2, B_3 were determined via GPS measurements and the distances A_1B_1, A_2B_2, A_3B_3, representing the slab's movement, were calculated. It must be pointed out here that the differences, between the calculated distances and the truly imposed ones, were of the range of ± 1*mm*, it is therefore concluded that GPS could be used for such a purpose.

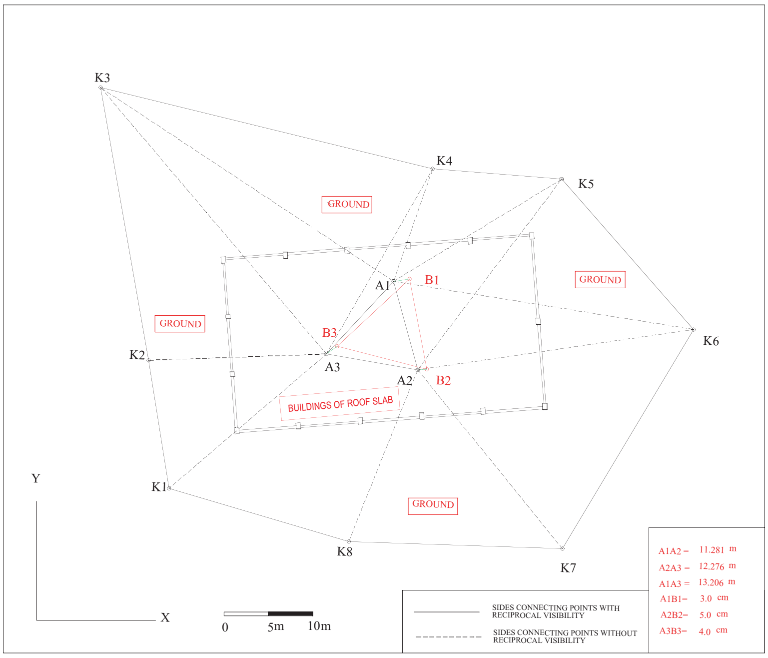

Figure 3. The control network for the determination of the horizontal movement of the building's roof slab.

The mean rotation angle φ of the slab was computed from the differences of the azimuths of the sides of the two triangles ($A_1A_2A_3$) and ($B_1B_2B_3$):

$$\varphi = \frac{1}{3}\sum_{i=1}^{3}\left(a_{B_iB_{i+1}} - a_{A_iA_{i+1}}\right) \tag{4.1}$$

From the point of intersection of the perpendiculars at the middles of the three displacement vectors (A_1B_1), (A_2B_2), (A_3B_3) the mean pole Π of the slab's shift was also determined. Thus the displacement D_i of any other point i of the slab can be computed through the equation:

$$D_i = \varphi^{rad} \cdot S_{i,\Pi} \tag{4.2}$$

where:

φ^{rad}: the rotation angle determined through (4.1) in rad, and

$S_{i,\Pi}$: the distance of point i from the pole Π.

4. Conclusions

From the methodology proposed for the detection of the divergences in precast construction, as applied in the above case study, the following conclusions are withdrawn:

- The use of geodetic methodology is a reliable tool for the detection and estimation of the divergences occurring during the various stages of precast construction. These divergences, estimated through the comparison of the coordinates of the control points of the geodetic network, are given in the local reference system of the network. They can therefore be compared to each other as well as to other important geometric characteristics of the construction.

- At the stage of the design of the geodetic control network the appropriate observation scheme as well as the instrumentation is selected. Therefore the quality measures of the network together with the time needed for the observations can be pre-estimated so that the network is optimized from the point of view of the maximum reliability of the final results and of the minimum total cost.
- The estimates of the adjustment (coordinates of the network's points) are free from the possible gross errors of the observations since appropriate statistical tests concerning the precision and reliability of the results, are applied. Moreover, the statistical testing of the significance of the estimated divergences ensures their reliability.
- Since a large number of the control points of the network is inaccessible, the use of reflectorless total stations of appropriate accuracy, for the measurement of the network's elements, permits the measurement of all the sides between the reference and the control points of the network. Therefore the network's scale will be stronger and the quality of the results better.
- The reference points of the network can be used for the setting out of the construction in order to ameliorate the accuracy in the positioning of the foundation precast elements. They should therefore be established before the beginning of the construction works, in safe and stable locations and their monumentation should be permanent.
- It would be very useful if the marking of the control points at the selected positions is done during the precast elements' fabrication in the industry.
- The geodetic control network established for the estimation of the divergences of a precast construction, can also be used for the monitoring of the construction's kinematic behavior, after its erection. This is especially important in areas where the seismic hazard is great. Therefore the stability of the reference points of the network should be ensured. For this purpose GPS measurements can be used where possible.

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