Height Systems for 3D Cadastres

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Key words: Height Definition, Height System, Quality Requirements

SUMMARY

3D systems are built assuming the existence of suitable height reference systems. However, the earth is neither homogeneous nor flat, the plumb lines are curved and not parallel and the definition of height and the implementation of geodetic height systems is complex. 3D cadastral systems have the same problem. Cadastral systems are designed to persist for centuries. Them the earth itself is changing and this needs to be considered in the system design.

The paper starts with a discussion of existing height systems and the determination of height. Afterwards quality requirements for height in a 3D cadastral system are determined using some typical examples. The examples range from a tunnel below the Alps (e.g., the Gotthard base tunnel) to apartments in an apartment building. Deviations must be considered when creating the height system. The paper gives some indication where problems may occur and which pitfalls should be avoided.

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1. INTRODUCTION

Discussions on 3D systems typically assume the existence of suitable height reference systems. Since the earth is neither homogeneous nor flat, the plumb lines—which we assume to be representations of the vertical axis—are curved and not parallel. This causes problems in the definition of height and the implementation of geodetic height systems. These problems affect all applications where height needs to be determined for large areas like whole countries.

3D cadastral systems comprise all problems of other 3D systems but additionally cadastral systems are designed to persist for centuries. During such periods the earth is changing and this needs to be considered in the system design. Classical 2D cadastral systems did not face this problem because::

1. 2D cadastral systems use an analogue representation in scales where the effects of drawing precision and line width exceed the change in reality.
2. 2D cadastral systems define the boundary line itself in reality and use the cadastral representation as an approximation only.

The first approaches are not applicable anymore since modern system should use the benefits of modern computer systems (Kaufmann and Steudler 1998). Depending on the level of detail used for the 3D representation, the second approach may not be valid either (Stoter and van Oosterom 2006, p. 58). Stoter and van Oosterom touched the question of height (Stoter and van Oosterom 2006, pp. 192-195) but they approached the problem mainly from the modelling perspective. The goal of the paper is to discuss the various aspects of the problem from both, a geodetic and a cadastral perspective and to provide a solid basis for further discussions.

Geodesists defined various systems for height definition: geopotential numbers, dynamic heights, orthometric heights, normal heights, and ellipsoidal heights (Hofmann-Wellenhof and Moritz 2005, pp. 157 ff.). Each of them has advantages and disadvantages. The paper starts with discussing these systems, their characteristics, and how the most commonly used height determination systems work. Then quality requirements for height in a 3D cadastral system are defined. The requirements differ between the various aspects: For example a tunnel below the Alps (e.g., the St. Gotthard tunnel) requires less precision in the vertical definition of the 3D parcel than an apartment within an apartment building, however, has a maxim tolerance of the thickness of the ceiling. These deviations must be considered when creating the height system. Considering all these aspects allows assessing the suitability of each height system for 3D cadastres.

2. DEFINITION OF HEIGHT

Height is usually defined as the "*vertical distance from a datum*" (Wolf and Brinker 1989, p. 109). This leads to several questions:

- Which measurement unit shall be used to determine the distance?
- Which datum shall be used?
- What does vertical mean?

These questions seem to be trivial at first but they are not. Meters and Gal are used as measurement units to determine height, the geoid, the pseudo-geoid, and the ellipsoid are used as a datum and the vertical line may be assumed to be straight or bent. These differences result from different approaches to define height.

There are several practical requirements for height systems:

1. Height (or height differences) shall be easy to determine.
2. Point heights shall be well defined and path independent, i.e., the height difference of a closed loop should be zero.
3. Correctional terms for observations shall be small such that they can be ignored for small areas.
4. Heights shall be free of hypothesis.
5. Heights shall be physically meaningful.
6. Heights shall be geometrically determined.

There are two simple methods to determine a height difference: Rod levelling and hydrostatic levelling. Both use the plumb line to determine the vertical direction. However, both methods contradict the second requirement that closed loops should have no height difference. Figure 1 illustrates this problem. The almost parallel lines are equipotential surfaces, i.e., points on these surface have the same potential energy. The plumb line is perpendicular to these surfaces and, since the surfaces are non-parallel, the plumb line is curved. Examples are drawn between A and A' and B and B'. The lowest of the equipotential surfaces is at sea level and is defined as the geoid. The irregular line represents the surface of the earth. Assume the height difference between the points A and B shall be determined as a length along the plumb line. The length between A and A' is different than between B and B'. Rod levelling would determine the height difference in small steps along the surface which results in value between these extremes. This could not happen if the second requirement holds.

A number of different systems have been proposed to fulfil the requirements. The most commonly used systems are

- geopotential height,
- dynamic height,
- orthometric height,
- normal height, and
- ellipsoidal height.

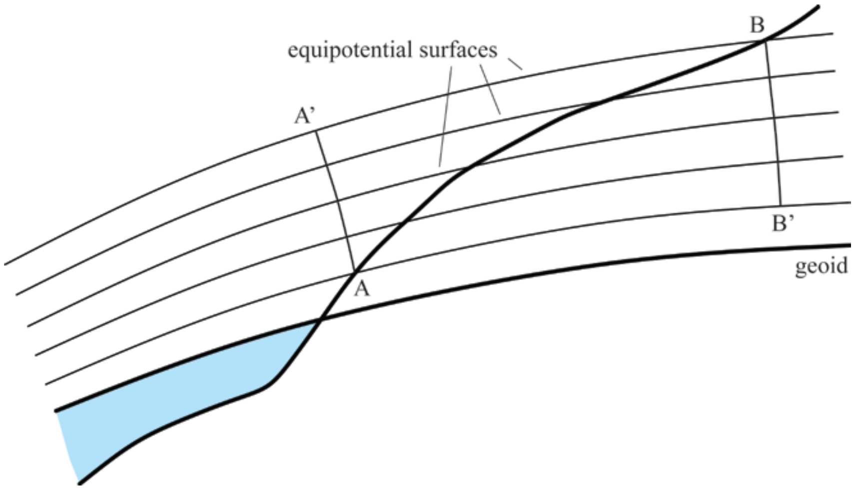

Figure 1. Non-parallelity of equipotential surfaces

2.1 Geopotential Height

Geopotential height or number *C* of a point is the potential difference of this point's potential and the geoids potential. As a potential difference, the geopotential number *C* is path-independent. It is the same for all points of the same equipotential surface. The geopotential number is typically measured in *kgal·m*. *gal* is a measure for acceleration and its SI-definition is cm/s^2. Although the potential does not describe a distance, it is the natural criterion for heights.

The disadvantages for geopotential heights are the difficult measurement process and the fact that it does not describe a distance. Also the determination is difficult. It requires a combination of geometrical levelling and gravity measurements. The levelling provides the distance and the gravity measurement the acceleration.

2.2 Dynamic Height

Dynamic heights are derived from geopotential numbers by

$$H^{dyn} = \frac{C}{\gamma_0},$$

where *C* is the geopotential number and γ_0 is normal gravity for an arbitrary standard latitude. Typically the normal gravity for the latitude of 45° is used:

$$\gamma_{45^\circ} = 9.806199203 ms^{-2} = 980.6199203 gal .$$

The division by γ_0 converts the geopotential number into length. Since it is a 'scaled' geopotential number it still has physical meaning, the determination is still path independent, and the measurement process is the same as for geopotential height. The use of an arbitrary value for the scaling keeps the system free of hypothesis on the mass distribution in the earth. Unfortunately, the dynamic height has no geometrical meaning. When translating vertical distances into difference of dynamic height, the correctional terms can become quite large. This makes dynamic heights useless for practical applications but they can still be used for the definition of national or continental height systems.

2.3 Orthometric Height

Orthometric height is defined as

$$H = \frac{C}{\bar{g}}$$

where C is the geopotential number and $\bar{g}$ is the average gravity along the plumb line between the geoid and the observed point. The determination of $\bar{g}$ is done by

$$\bar{g} = \frac{1}{H}\int_0^H g(z)dz \ .$$

This would require detailed knowledge of the mass distribution within the earth along the path to determine *g(z)*. Since the distribution is not known, a variety of approximations has been developed. The simplest version is the Prey reduction (Hofmann-Wellenhof and Moritz 2005, p. 139)

$$g(z) = g + 0.848(H - z) \ .$$

$\bar{g}$ is then the arithmetic mean of the gravity g observed at the surface and the gravity g_0 computed at the corresponding geoidal point:

$$\bar{g} = \frac{1}{2}(g + g_0) \ .$$

Orthometric heights have several practical advantages:

- Point heights are well defined and path independent,
- they define a geometrical property, and
- corrections of observed values are small.

The size of the corrections allows ignoring them completely for local measurements. Thus local measurements only require geometrical levelling and still produce orthometric heights. This made orthometric heights the best option for height systems in the 20th century for a large number of countries. However, orthometric heights induce some problems, too. Water may flow between points with the same orthometric height because it is not guaranteed that the geopotential number is equal. Another problem is that orthometric heights are not free of hypothesis.

2.4 Normal Height

The height systems described above use the geoid as a reference surface. Normal heights use an approximation of the geoid, an ellipsoid. The ellipsoid is a mathematically more convenient reference surface than the geoid.

The basic assumption for normal heights is that the earth has a regular gravity field so that the potential *W* of the geoid is equal to the potential *U* of the regular gravity field. The potential difference is then

$$W_0 - W = C = \int_0^{H^*} \gamma dH^*$$

with the gravity γ at the ellipsoid and normal heights H* can then be defined as

$$H^* = \int_0^C \frac{dC}{\gamma} \ .$$

A given point B on the surface of the earth has a specific potential W_B and a specific normal potential U_B. These two potentials are different because all irregularities of the gravity field of the earth are ignored in the regular gravity field. There is a point *B'*, however, where the normal potential $U_{B'}$ equals the true potential $W_{B'}$. The normal height H_{B*} of the point *B* is then the orthometric height of *B'* on the ellipsoid using the regular gravity field.

No hypothesis is required for this definition since a regular gravity field is used. Other advantages are the independence of path and the small corrections. However, the height has no physical meaning and it is not geometrically determined.

2.5 Ellipsoidal Height

Ellipsoidal heights have a simple geometric definition: The height is the straight-line distance along the surface normal of the ellipsoid. Obviously they have no physical meaning but the determination of height is path-independent and no hypothesis is required. However, the computation of heights from corrections can require large corrections.

The importance of ellipsoidal heights lies in the fact that GNSS like GPS heights are ellipsoidal heights. Thus they are easy to determine with common GNSS-receivers. Such equipment is already in use for 2D surveys. However, the height component is typically of lower quality than the plane coordinates.

2.6 Summary and Conclusions

The overview showed that definition of height is neither simple nor straightforward. There is no perfect height definition that produces physically meaningful heights that are easy to measure and do not require correction of the observations.

Conversion between the different systems is not trivial either. The different reference surfaces as well as the curvature of the plumb lines need to be addressed. The vertical difference between geoid and ellipsoid is called geoid undulation and is up to almost 200 m when using a reference ellipsoid. Deriving orthometric heights from ellipsoidal heights, for example, requires knowledge of the geoid undulation. This requires a precise knowledge of the geoid itself. One of the currently best global models is the GRACE Gravity Model 3 (Tapley et al, 2007). The determination of the geoid undulation in Croatia produces an accuracy of approximately 10 cm (Grgić et al, 2009). This corresponds to current quality, which is estimated to 10 cm for distances of 10 to 20 km and 30 cm for a 1° resolution (~111 km). Thus a combination of GNSS height observations with terrestrial observations creates uncertainties in the range of decimetres.

3. DETERMINATION OF HEIGHT

The determination of height uses measurement equipment that can observe a specific quantity, e.g., length or gravity. The geometrical determination of height is typically the easiest: A horizontal plane is used as an approximation for points of equal height and then vertical deviations to that plane are observed. This is the principle of geometrical levelling as shown in Figure 2. The drawback of geometric levelling is that it does not fit strictly to any of the height systems defined in section 2 because the horizontal plane is only an approximation of the reference surface. Thus for large levelling networks gravity must be observed along the levelling routes. Still, geometrical levelling is used to determine local height differences for orthometric and normal height systems.

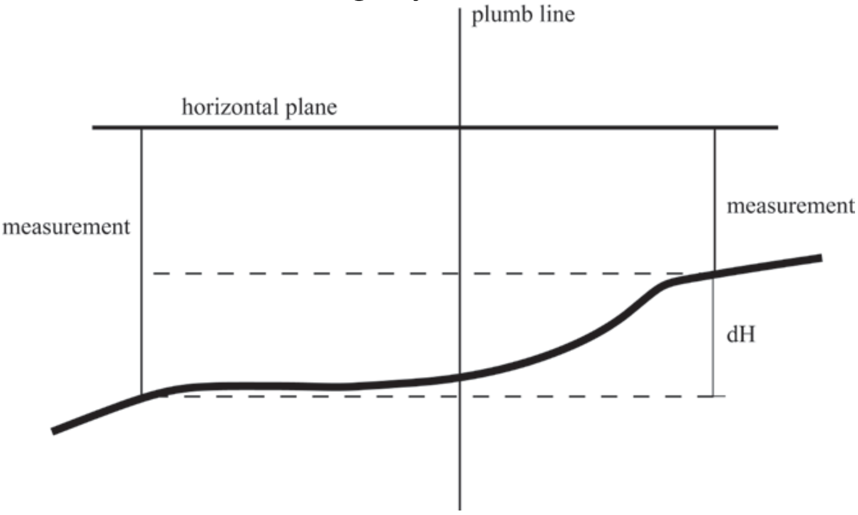

Figure 2. Geometric levelling

The determination of ellipsoidal heights must refer to the surface normal of a specified ellipsoid. Observing this surface normal in reality is impossible except for very specific points where by definition surface normal of the ellipsoid and plumb line coincide. Otherwise the direction can only be determined mathematically. Thus absolute ellipsoidal heights are typically derived from measurements to known points. GNSS, for example, observe the distance to known satellite positions and then compute the position of the receiver in a geocentric coordinate system. These coordinates can then be used to determine ellipsoidal latitude, longitude, and height. Local observation of ellipsoidal height difference is not possible but geometrical levelling can be used to estimate ellipsoidal heights if the deflexion of the vertical is known in the area of interest.

The practical advantage of geometrical levelling is that it is applicable everywhere. Geometrical levelling is possible outside as well as inside of buildings or tunnels and thus is can provide height information where satellite-based measurement methods like GPS fail. In addition, height determination is possible with high precision if required, which is still a problem of GNSS.

4. RELATIVE VS. ABSOLUTE HEIGHT DEFINITION

Height can be defined either in absolute terms or in relation to the height of another point. Geopotential numbers are an example for absolute heights. Strictly speaking, all other height systems discussed in section 2 are relative systems since they relate height to a specified shape, the geoid or an ellipsoid. However, this is typically not meant by ‘relative height definition’. Usually statements like “5 m below the ground” are seen as relative measurements. These values are much easier to determine than absolute values since—in most cases—they can be observed directly. If the distance becomes too big, relative heights may be difficult to determine, e.g., how deep below the ground is the Gotthard Base Tunnel at a given point? The answer is not only varying with the position in the tunnel, it is also difficult to determine.

Stoter and van Oosterom propose the use of absolute height coordinates (Stoter and van Oosterom 2006, p. 192f). A challenge for absolute height coordinates, however, is the fact that the earth is not static. Scandinavia, for example, raises with a speed of up to 1 cm/year (Lidberg et al, 2007). Height coordinates should change with the same rate. However, they typically have fixed values. The system only works in practice because the coordinates are derived locally using reference points and these points move together with the surface. Thus, although the absolute coordinates provide incorrect values, the difference between the coordinates is correct due to the high spatial autocorrelation of the errors.

In everyday life local height systems are more common. For example, building permits restrict the height of buildings and the level of groundwater is also expressed as “in a depth of 3 m”. In many cases relative measures may not use length units to specify the height. “My apartment is in the second floor” is a relative height definition that is often used and easy to check by observation. However, in terms of metres it may correspond (in Vienna) with a value between 4 and 20 m. It should be noted, though, that even this simple method contains pitfalls. Counting may start with zero (ground floor) or one (first floor) and legal regulations may lead to a situation like in Vienna where between ground floor and first floor many old houses have an intermediate storey, a mezzanine, or both.

A problem for relative height definition with respect to the surface of the earth is that this surface may change. The reason for a height change may be a natural phenomenon, e.g., a land slide (compare Figure 3). It can also occur because of human actions, e.g., constructions. Road reconstruction, for example, may change the vertical position of the road either to minimize vertical movement or because a new base layer had to be created.

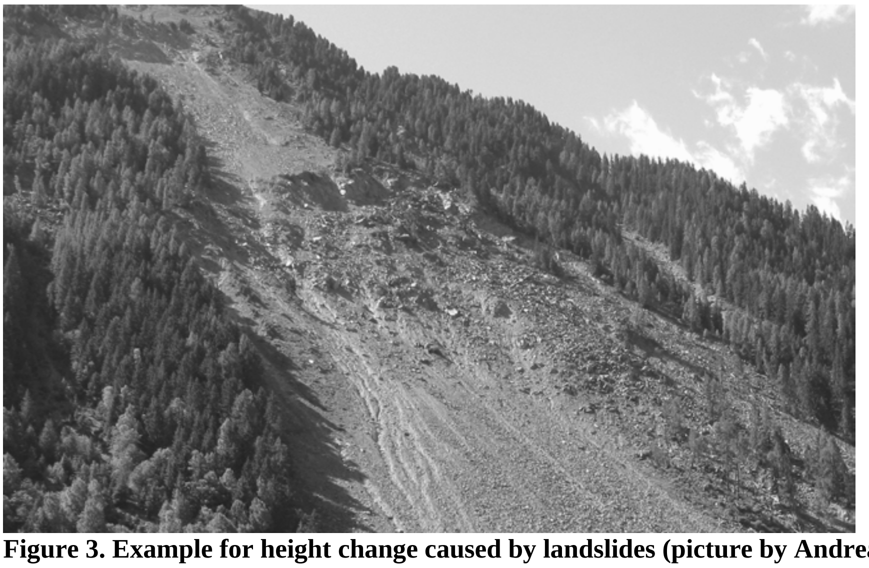

Figure 3. Example for height change caused by landslides (picture by Andreas Eichhorn, FWF project KASIP, P20137)

5. QUALITY REQUIREMENTS FOR HEIGHT IN 3D-CADASTRES

The height definition in the cadastre has to fulfil at least the accuracy requirements of the cadastre. However, since the discussion on 3D cadastres emerged in the end of the 20th century, the quality requirements of the cadastre are still discussed There is no practical experience with lawsuits and thus there may be opinions expressed by lawyers but there are no valid legal conclusions yet.

The quality requirements for plane coordinates could be used as an indication for the quality requirements. Coordinates in Austria are registered with centimetre precision. This corresponds with the geometrical quality of average construction work like placing walls. Precision for boundaries of farmland can be less accurate since boundaries of farmland need to be known with a precision of approximately 20 cm (less than the width of a plough blade). For land owners the shape of the parcel is more important than the absolute geographical position. The shape determines possible uses, e.g., if a specific building can be created while complying with legally defined clearances to the boundary. This perspective is also used when a court must decide if clearances are met (Navratil 2008). There are thus 2 conclusions that could be drawn on the quality of plane cadastral coordinates:

- quality requirements for a cadastre are not uniform throughout the country;
- local accuracy is more important than global accuracy.

Both conclusions may also be true for the vertical dimension in 3D cadastres. Four examples described below give an impression on the varying needs for quality:

5.1 Tunnel deep below the surface

The Gotthard Base Tunnel mentioned in section 4 is an example for a tunnel that is deep below agricultural land. There is so much vertical distance between the tunnel and the surface that the existence of the tunnel does not influence the land use above. The only perceptible effect could be vibration caused by trains. Therefore, registration of the tunnel could be useful to avoid subsequent legal trouble. The accuracy required for this tunnel is not high. For example, if the tunnel is 100 m below the surface, then an accuracy of 5 to 10 m should be sufficient, i.e. 5 to 10% of its depth. In this range of accuracy, changes on the surface itself do not provide a problem. Even land slides with vertical effects of several metres do not falsify the information significantly.

5.2 Tunnel close to the surface

Tunnels close to the surface induce more problems. Figure 4 shows an example of a road tunnel that is only a few metres below the surface. The area above the tunnel, behind the fence, indicated by the arrow, is used as playground. Limited construction work, e.g., to build a climbing frame, is possible. In this case information on the depth of the tunnel may be essential to avoid damage to the tunnel structure during the creation process.

Figure 4. Example for a road tunnel close to the surface (Hackl 2007)

However, such tunnels are also built below cities, e.g., for the metro. In these cases the land above the tunnel is typically used for buildings. If the tunnel is built after the buildings on the ground then detailed knowledge is necessary on the buildings. Vienna, for example, has a large number of old buildings with up to 3 basement levels. Depending on the rock structure, this requires a vertical clearance distance for any construction work. The same clearance distance must be considered if creating a building on top of the metro. These vertical clearances are usually rounded to metres. Height information must be more precise to avoid problems. A 3σ-accuracy of at maximum 0.5 m is recommendable, which then leads to an accuracy of 16 cm.

In built environment the problem of vertical surface change for relative heights is problematic. Cities are constantly renewed and thus the absolute vertical position of the surface changes. Even if there is only a new coat of tarmac put on a road, the vertical change will be a few centimetres. Thus relative heights require thorough planning of update processes in cities.

5.3 Construction above public space

Similar problems occur if constructions extend into the space above public areas like streets. In such cases, clearances must be met, too. Roads, for example, may have height limits for cars and it must be ensured that cars that comply with these limits can use the road without any problems. Absolute height is irrelevant as long as the relative clearance is met. In contrast to underground constructions, however, checking the compliance of such constructions with vertical clearances is easier. Legal decision making does not consider measurement quality (Twaroch 2005). Thus the quality requirements must be high enough that problems with lawsuits are avoided.

5.4 Ownership of apartment

Ownership of apartment in 3D cadastres allows separation of spaces owned by different persons within a building. Usually this separation is vertically. Thus the border must lie within the ceiling. The assumption of a ceiling thickness of 30 cm leads to a vertical accuracy of at most 5 cm.

The easiest way would be counting the floors, e.g., ground floor owned by X, first floor owned by Y, and second floor owned by Z. This is getting difficult in cases where the separation is not so strict, e.g., in the case of an apartment building with 3 apartments shown by Stoter and van Oosterom (Stoter and van Oosterom 2006, p. 41). Other problems occur if buildings are placed along hillsides and it is not clear where to start counting the floors.

Absolute heights may be difficult in this situation. If the building subsides, absolute heights may lead to a situation where the boundary moves out of the ceiling. The legal consequences of such an incident are not clear.

6. DISCUSSION AND CONCLUSIONS

The examples showed that the quality demands vary dramatically from a few centimetres to a dozen metres. This must be addressed by any 3D cadastre because otherwise the highest demands must be used to stipulate quality requirements and this will lead to unnecessarily expensive surveys. The examples also showed that relative height information is more important than absolute height information. This may, however, cause specific problems: Assume a metro running directly underneath a street. This situation has some advantages:

- there is less pressure on the tunnel compared to a situation where a building is located above the tunnel;
- construction is easier because it may even be possible to create the metro with the trench method;
- metro and road network may be owned by the same organisation.

However, roads may have larger inclinations than metro lines. Thus, the vertical distance between metro line and road surface will vary. Absolute vertical coordinates would show that the metro is smooth. Relative heights, however, would show a different behaviour. The example also shows that a single height for the object is not sufficient because the legal object may have a large horizontal extent even if it is always cut on the edges of the 2-dimensional parcels.

A practical problem is obtaining the height of the surface and the legal objects that shall be registered. Terrain height is nowadays typically obtained by Airborne Laser Scanning, ALS, (Kraus and Pfeifer 2001) and height determination of objects is often performed with GNSS receivers. Both kinds of systems create ellipsoidal heights. National height reference frames, however, are frequently based on more physically defined height systems and the conversion between these systems is not simple. This may lead to practical problems when implementing 3D cadastres. Tunnels can usually not be measured with these methods although some tunnels are built with the trenching method and then GNSS and ALS can be used. However, this is only a problem for built-up areas.

It can be concluded that there is no optimum height reference system. Small and flat countries may be able to use any reference system whereas other countries must take existing data into account and the height system used for them. The question whether absolute or relative heights should be used is also difficult to answer. Many practical questions require relative information only. The problem is keeping track of height changes of the surface. Using the difference between absolute heights to determine relative height information would be possible but quality aspects need to be checked first. However, it also may be necessary to use different height systems in parallel and store values in different systems with one cadastral object.

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BIOGRAPHICAL NOTES

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