

# 2. Grid modeling: Filtering: Why? Huge amount of points (e.g. Kongsberg EM3002) 40 Hz 500 pts./swap 20.000 pts./sec. or 72 million pts./hour + interpolation by multibeam system software to equidistant

grid

=> More or Less points depending on grid interval distance

# 2. Grid modeling: Filtering: How?

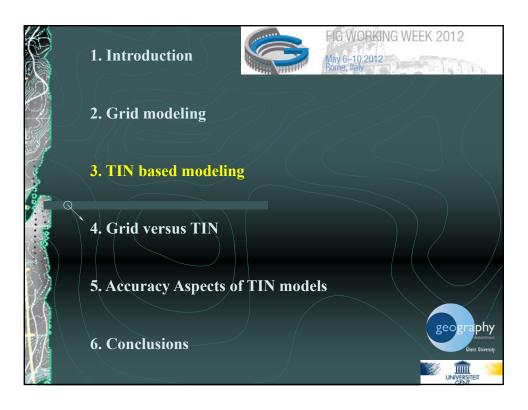
• Modify grid interval distance

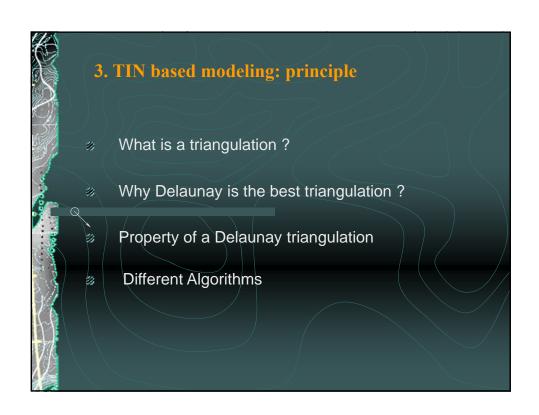
e.g. 1 by 1 m => 5 by 5 m

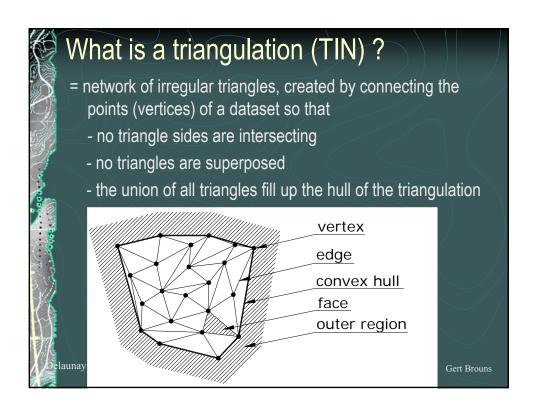
Reduction by 96 %

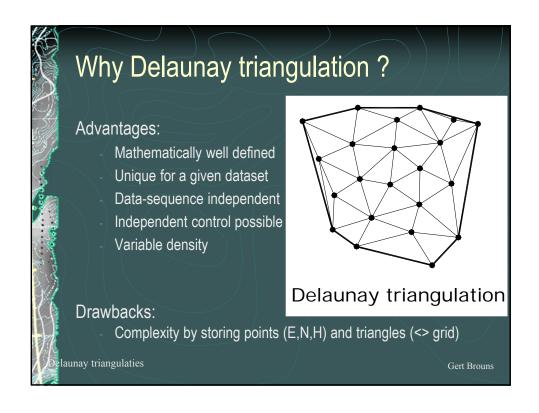
Loss of resolution can cause loss of seabottom details

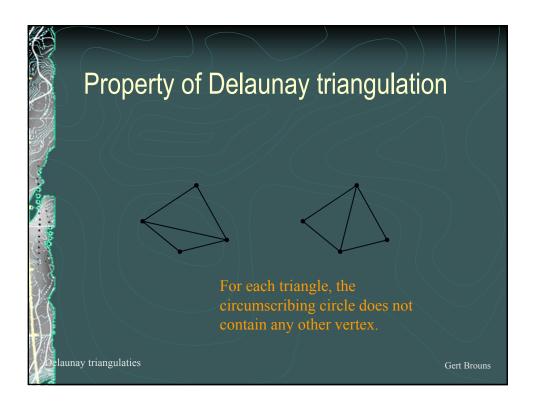
- Use of "smarter" algorithms
  - Depth is weighted average of all depths of initial cells
    - Weighting factor = inverse distance to power n (2-?)
  - Minimum depth (control survey, not for volume computat.)
  - Use model with variable grid intervals => complex

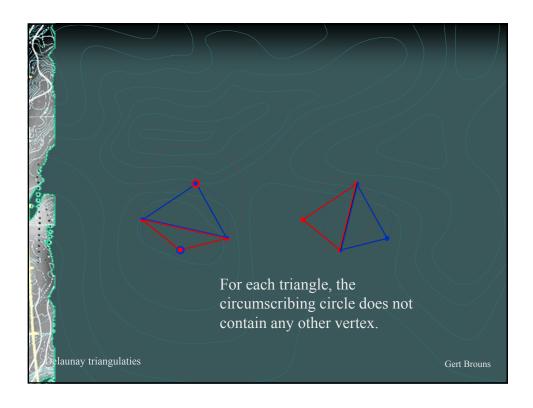


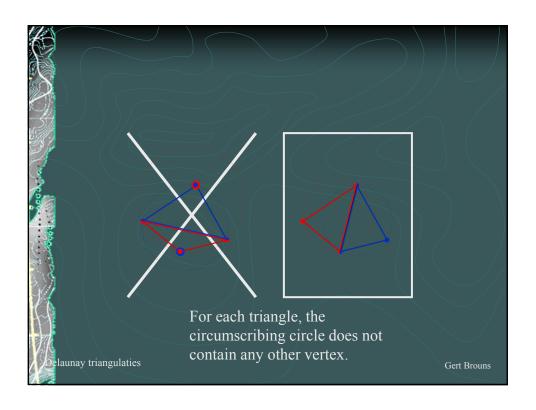


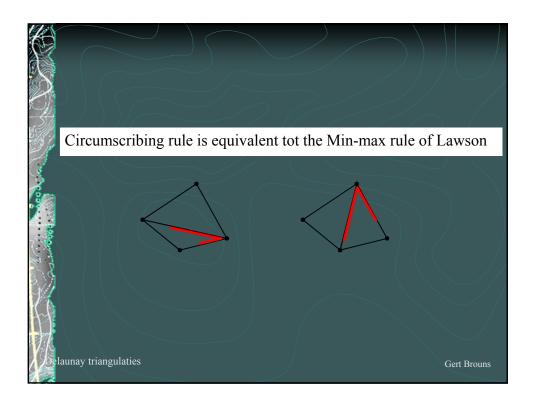


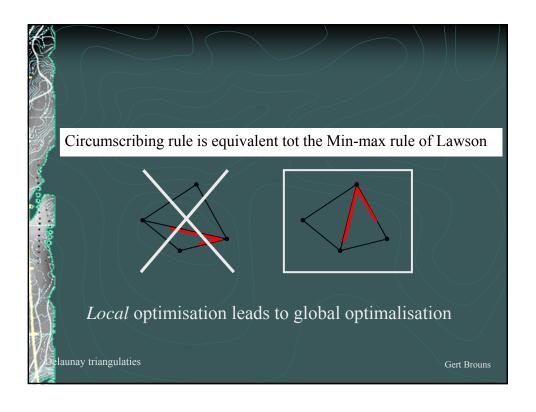


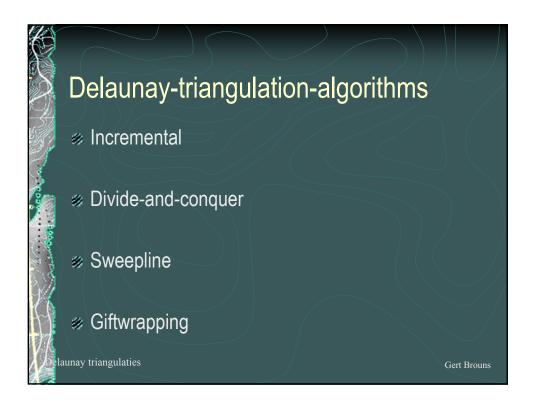


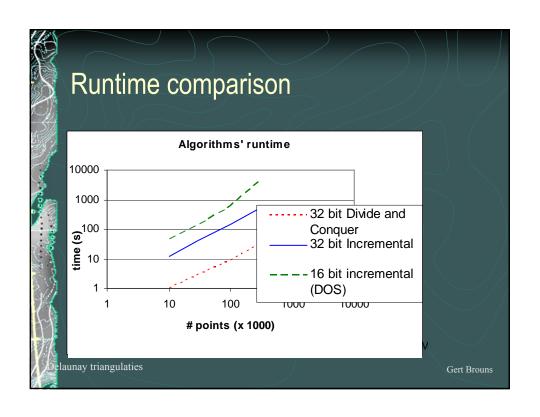


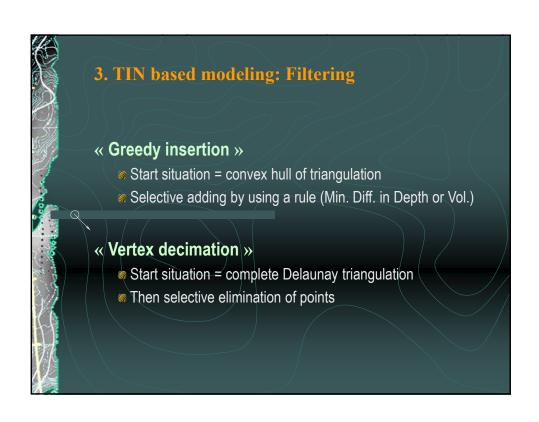


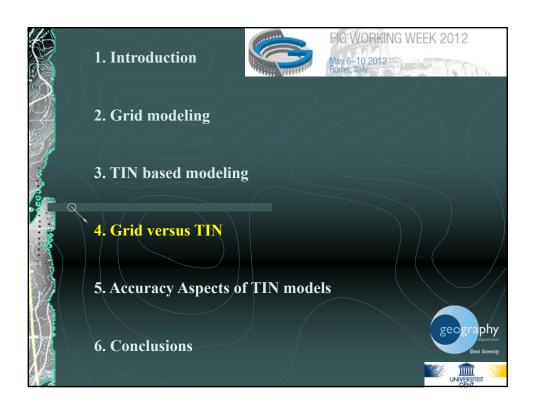


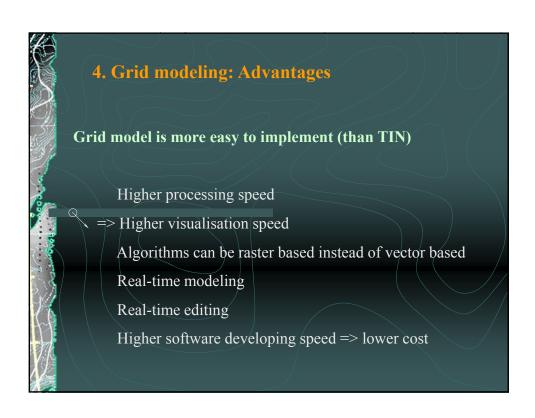


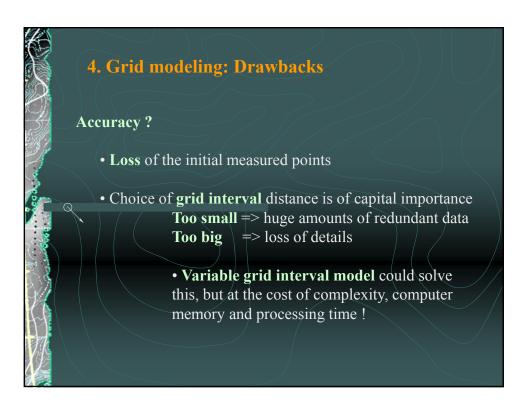


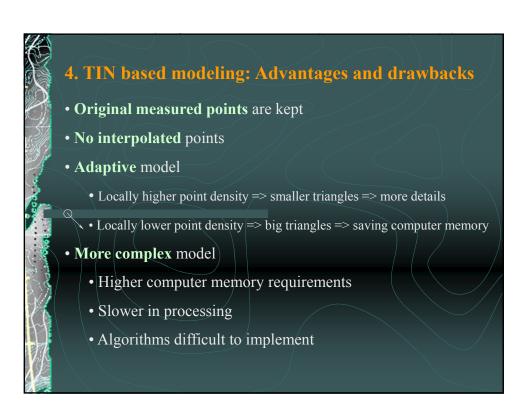


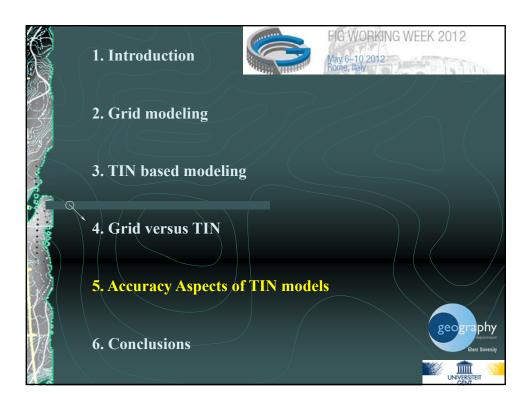


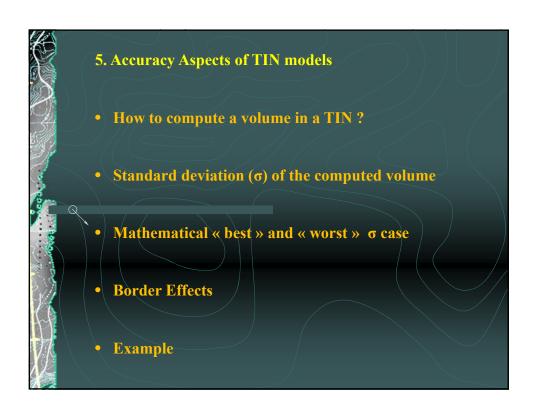


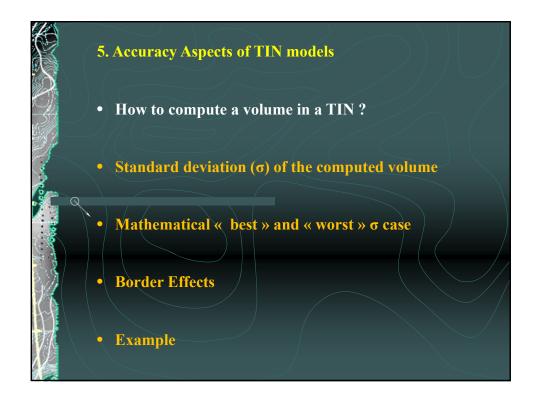


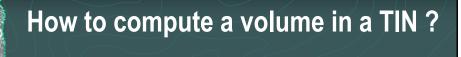






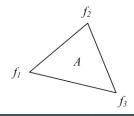






With  $\underline{A_j}$  as the <u>planimetric</u> surface of a triangle j,  $f_{ref}$  as the height of the horizontal reference plane and  $f_i$  as the elevation of the 3 vertices i of the triangle, the volume  $V_j$  generated by one triangle j is equal to

$$(\frac{1}{3}(f_1+f_2+f_3)-f_{ref})A_j=V_j$$



# How to compute a volume in a TIN?

The total volume V is the sum of the volumes of all individual prisms, thus

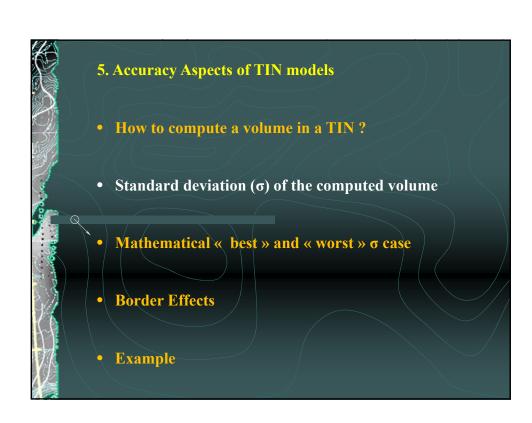
$$\frac{1}{3} \sum_{i} f_{i} \left( \sum_{f_{i} \in A_{j}} A_{j} \right) - f_{ref} A_{tot} = V$$

If we call  $B_i$  the sum of the surfaces of all triangles with point i as vertex or

$$B_i = \left(\sum_{f_i \in A_j} A_j\right)$$

Then we can write

$$\frac{1}{3}\sum_{i=1}^{n}f_{i}B_{i}-f_{ref}A_{tot}=V$$



Assuming that all  $f_i$  are independent, the variance of the volume can be found

$$\frac{1}{3} \sum_{i=1}^{n} f_i B_i - f_{ref} A_{tot} = V \quad \Longrightarrow \quad Var(V) = \frac{1}{9} \quad \sum_{i} Var(f_i) B_i^2$$

The standard deviation and variance  $\underline{Var(f_i)}$  of the elevation of a point is usually assumed to be constant so that, with n the total number of points

$$Var(V) = \frac{Var(f)}{9} \sum_{i} B_{i}^{2}$$

$$\sigma(V) = \frac{\sigma(f)}{9} \sum_{i=1}^{n} B_i^2$$

This form is useful in the case of a TIN model based on non-equidistant points.

$$\sigma(V) = \frac{1}{\sqrt{n}}.\sigma(f).A_{tot}.\sqrt{1 + \left(\frac{\sigma\left(B\right)}{\overline{B}}\right)^{2}}$$

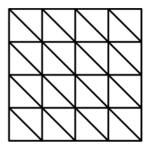
The latter form is applicable to TIN's of irregular spaced points but is also particularly suited in the case of a TIN model based on equidistant points.

TIN with regular spaced points

$$\sigma(V) = \frac{1}{\sqrt{n}} \sigma(f) A_{tot} \cdot \sqrt{1 + \left(\frac{\sigma(B)}{\overline{B}}\right)^2}$$

Assuming a TIN of regular spaced <u>points</u>, and without the consideration of border issues, a minimum of the standard deviation can be found for a layout where all rectangular cells of the TIN have an identical direction of the diagonal. In this case, every non-border point has 6 neighboring triangles and as all triangles have the same surface, g(B) = 0, and it can be found that

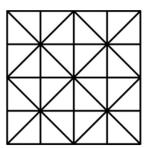
$$\sigma(V) = \frac{1}{\sqrt{n}}.\sigma(f).A_{tot}$$





In case the diagonals in the grid system are alternating, half the number of non-border points have 4 neighbors and the other half 8.

$$\sigma(V) = \frac{1}{\sqrt{n}}.\sigma(f).A_{tot}.\sqrt{1 + \left(\frac{\sigma(B)}{\overline{B}}\right)^2}$$



Hence

$$\frac{\sigma(B)}{\overline{B}} = \frac{2}{6} = \frac{1}{3}$$

and

$$\sigma(V) = \frac{1}{\sqrt{n}}.\sigma(f).A_{tot}.\frac{\sqrt{10}}{3}$$

The difference in standard deviation of the volume between the optimal layout and the worse case layout is only a factor  $\frac{\sqrt{10}}{3}$  or 5.4 % difference.



- 5. Accuracy Aspects of TIN models
- How to compute a volume in a TIN?
- Standard deviation (σ) of the computed volume
- Mathematical « best » and « worst » σ case
- Border Effects
- Example

# Mathematical « best » and « worst » σ case

$$\sigma(V) = \frac{1}{\sqrt{n}}.\sigma(f).A_{tot}.\sqrt{1 + \left(\frac{\sigma(B)}{\overline{B}}\right)^2}$$

The mathematical "minimal variance" solution is obtained when all n surfaces  $B_i$  are equal,

$$Var_{\min}(V) = \frac{Var(f)}{n} A_{tot}^{2}$$

$$\sigma_{\min}(V) = \frac{\sigma(f)}{\sqrt{n}} A_{tot}$$

The mathematical theoretical "maximal variance" solution for the volume is when one surface  $\underline{A_i}$  is maximal  $(\underline{Aj} = \underline{A_{tot}})$  and all other  $\underline{A_k}$  are neglectable and therefore set equal to zero. In this case there are 3 non-zero  $B_i = A_j = A_{tot}$ , and

$$\sum_{i} B_{i}^{2} = 3A_{tot}^{2}$$

$$Var_{max}(V) = \frac{Var(f)}{9}.3A_{tot}^{2}$$

$$\sigma_{max}(V) = \frac{\sigma(f)}{\sqrt{3}}.A_{tot}$$

## 5. Accuracy Aspects of TIN models

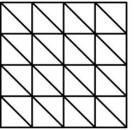
- How to compute a volume in a TIN?
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# **Border Effects**

$$\sigma(V) = \frac{1}{\sqrt{n}} . \sigma(f) . A_{tot} . \sqrt{1 + \left(\frac{\sigma(B)}{\overline{B}}\right)^2}$$

### Border effects

Assume a geometric layout with 6 neighbors and a total of n points, m (out of the n) points are border points, meaning that they are laying along the edges of the triangulated zone, within the area where the volume is computed, and where the ration m/n is called  $\varphi$ , with  $0 \le \varphi \le 1$ .



Finally

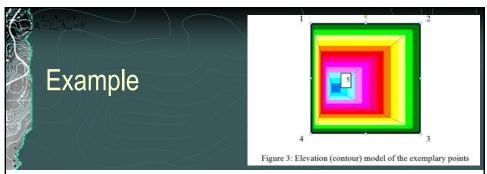
$$\sqrt{1 + \frac{\sigma(B)^2}{\left(\overline{B}\right)^2}} = \sqrt{1 + \frac{9\varphi(1 - \varphi)}{\left(6 - 3\varphi\right)^2}} \approx \sqrt{1 + \frac{\varphi}{4}} \approx 1 + \frac{\varphi}{8}$$

As  $0 \ll \phi \ll 1$ , the maximal border effect on the standard deviation of the volume is an augmentation of the standard deviation with 1/8 or 12.5 %.

Usually  $\varphi$  is close to 0 and the border effect on the standard deviation on the volume is neglectable.

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# 5. Accuracy Aspects of TIN models How to compute a volume in a TIN? Standard deviation (σ) of the computed volume Mathematical « best » and « worst » σ case Border Effects Example



The following small data set with 5 irregular spaced points given in (E, N, H) is considered:

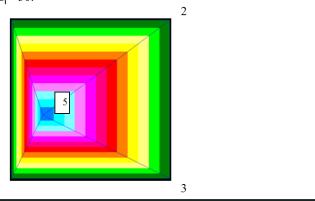
E	N	Н
0	0	0
0 5 0 5	0	0 0
0	5	0
5	5	
1	2	6

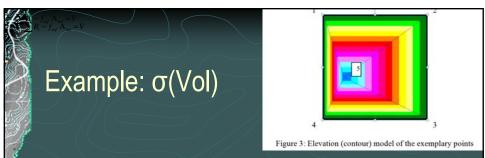
Table 1: coordinates of the example

A standard deviation of 0.5 for each of the height values is given.



The volume between a zero-level reference and this surface is easily computed as: 1/3 \* planimetric surface \* prism height. In this example, where the prism height is equal to 6, the prism volume = 1/3 \* 5 \* 5 \* 6 = 50.





The variance of the volume is  $Var(V) = \frac{1}{9} \sum Var(f_i)B_i^2$ .  $Var(f_i)$  is computed as the square of the standard deviation of the heights or  $0.5^2 = 0.25$ . The five different  $B_i$  for point 1, 2, ..., 5 are

 $\begin{array}{lll} B_1 = 0.5*5*1+0.5*5*3 & = 10 \\ B_2 = 0.5*5*3+0.5*5*4 & = 17.5 \\ B_3 = 0.5*5*4+0.5*5*2 & = 15 \\ B_4 = 0.5*5*2+0.5*5*1 & = 7.5 \\ B_5 = 0.5*5*1+0.5*5*3+0.5*5*4+0.5*5*2 & = 25 \end{array}$ 

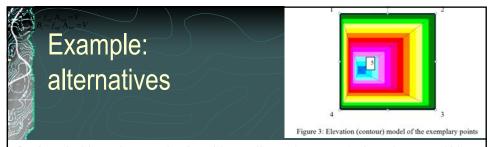
As a check, the sum of the  $B_i$  is always 3 times the total surface (75 = 25\*3).

The sum of the  $B_i^2$  is 100+306.25 +225 +56.25 +625 = 1312.5.

Hence the Var(V) is 0.25 \* 1312.5 / 9 = 36.458.

The standard deviation for the volume of 50 is the root of 36.458 or approx. 6.038.

Thus, the volume between the zero-level and the prism surface is 50 +/- 6.038.



If point 5 had been the central point with coordinates (2.5, 2.5, 6), the volume would have been the same.  $B_1$ ,  $B_2$ ,  $B_3$  and  $B_4$  would be all equal to 0.5\*5\*2.5+0.5\*5\*2.5=12.5,  $B_5$  being equal to 25.  $\underline{Var}(V)$  is then  $0.25*(4*12.5^2+25^2)/9=34.722$  and the standard deviation is now slightly reduced to 5.893 (instead of 6.038).

The theoretical "lower limit case" would yield  $\sigma_{min}(V) = \frac{\sqrt{(0.25*25^2/5)}}{5} = 5.590$ .

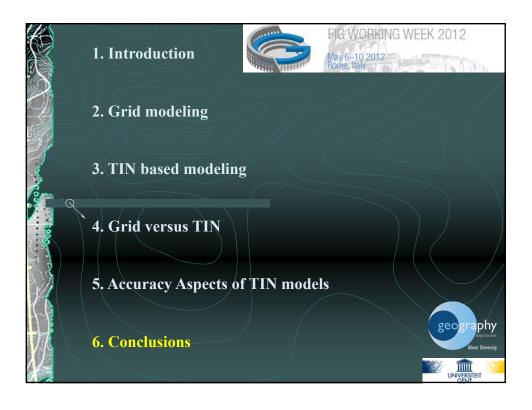
The theoretical "upper limit case" would yield  $\sigma_{max}(V) = \frac{sqrt}{(0.25*25^2/3)} = 7.217$ .

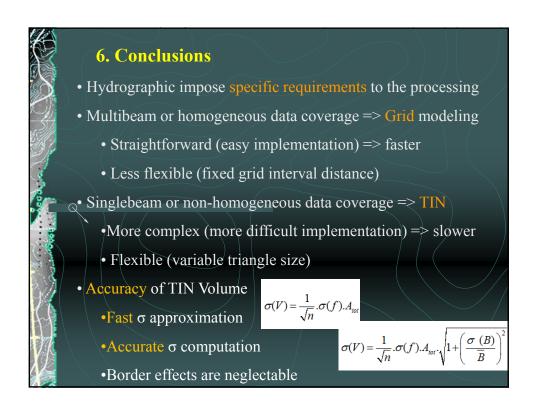
However, these two latter cases are purely hypothetical cases, both layouts being geometrically impossible.

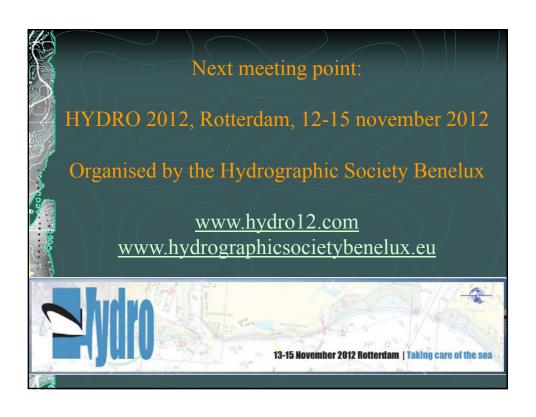


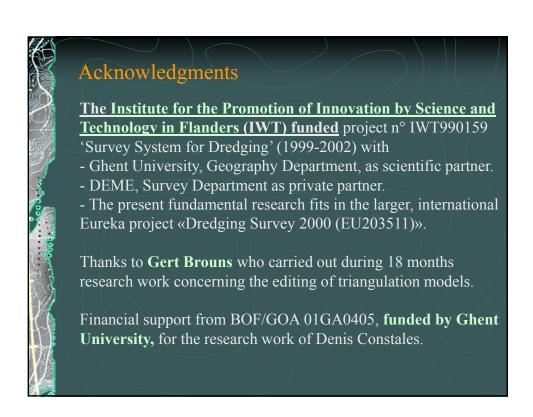
Conclusion:  $\sigma$  spread (< 10%) is small: "Best case" is good approximation and easy to compute!

 $\sigma(V) = \frac{1}{\sqrt{n}} . \sigma(f) . A_{to}$ 











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