

1 **Unified Multi–Sensor Advanced Triangulation (UMSAT) for System**
2 **Calibration and Trajectory Enhancement of Imaging and Ranging Sensors**
3 **Onboard Mobile Mapping Systems**

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8 **Key words:** mobile mapping systems, camera LiDAR integration, triangulation, system
9 calibration, trajectory enhancement

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12 **SUMMARY**

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14 Mobile mapping systems (MMS) such as uncrewed aerial vehicles (UAVs) and wheeled
15 platforms are widely used for a variety of applications such as precision agriculture, coastal
16 monitoring, digital forestry, transportation management, infrastructure monitoring, bulk
17 material estimation, and archaeological documentation. MMS are usually equipped with
18 integrated global navigation satellite systems/inertial navigation systems (GNSS/INS) as well
19 as imaging (e.g., RGB, multispectral, and hyperspectral cameras) and ranging (e.g., LiDAR)
20 sensors. UAVs are becoming a viable alternative for small area mapping due to their ease of
21 deployment, low cost, ability to fill an important gap between aerial and proximal mapping
22 platforms, miniaturization/improvement of GNSS/INS georeferencing technologies, and
23 proliferation of imaging/ranging sensors operating in different portions of the electromagnetic
24 spectrum.

25 Integration of image and LiDAR data can provide a comprehensive 3D model of the area of
26 interest. For such integration, ensuring a good alignment of derived products from single or
27 several platforms is critical. Although many works have been conducted on this topic, there is
28 still a need for a rigorous integration approach that minimizes the discrepancy between imagery
29 and LiDAR data/products caused by inaccurate system calibration parameters and/or trajectory
30 artifacts. This study proposes a tightly–coupled camera/LiDAR integration workflow for UAV
31 and wheeled remote sensing systems aided by a GNSS/INS unit. More specifically, the paper
32 presents a unified multi–sensor advanced triangulation (UMSAT), which can handle point,
33 linear, and areal features derived from imaging and ranging remote sensing systems aided by
34 GNSS/INS position and orientation unit. Through UMSAT, a general environment for system
35 calibration and/or trajectory refinement will be explored for improving derived data/products
36 from imaging and ranging remote sensing systems while focusing on transportation–related
37 datasets. Experimental results from real datasets will be presented together with
38 recommendations for future research to improve the performance of UMSAT in GNSS–
39 challenging, and potentially GNSS–denied, environments.

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Unified Multi–sensor Advanced Triangulation (UMSAT) for System Calibration and Trajectory Enhancement of
Imaging and Ranging Sensors Onboard Mobile Mapping Systems (12226)
Tian Zhao and Ayman Habib (USA)

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47
48 **1. Introduction**

49 Utilization of remote sensing technologies is becoming the norm for many applications due to
50 their ability to map large areas in a short time at a reasonable cost. More specifically, the
51 emergence of passive and active remote sensing modalities operating in different portions of
52 the electromagnetic spectrum allows for the derivation of a rich set of information useful for
53 various applications. Improvements and lower–cost of direct georeferencing technologies – i.e.,
54 integrated Global Navigation Satellite Systems/Inertial Navigation Systems (GNSS/INS) –
55 enable control–free mapping. In spite of the improving capabilities of spaceborne and airborne
56 remote sensing platforms, they do not provide reasonable spatial/temporal resolution at an
57 affordable cost. Modern mobile mapping systems (MMS) – such as uncrewed aerial vehicles
58 (UAVs) and wheeled systems – have emerged as promising platforms (Guan et al., 2014; Nex
59 & Remondino, 2014). Using these systems is motivated by their low cost, ease of deployment,
60 high maneuverability, and fine spatial/temporal resolution mapping. Recent availability of
61 miniaturized sensing and direct georeferencing units facilitates the use of such platforms in a
62 wide range of applications, such as precision agriculture, coastal monitoring, digital forestry,
63 transportation management, infrastructure monitoring, and archaeological documentation.

64 Imaging sensors including RGB and multispectral/ hyperspectral (MS/HS) cameras are widely
65 used. The majority of RGB cameras are based on a frame imaging mechanism, providing
66 imagery in a two–dimensional (2D) raster data structure with spectral information. With recent
67 developments in Structure from Motion (SfM) algorithms (Westoby et al., 2012), one can
68 generate dense point clouds. However, this reconstruction is contingent on adequate
69 overlap/side–lap among neighboring images and establishing sufficient matches. On the other
70 hand, most HS cameras are based on push–broom technology – also known as line cameras –
71 which capture 1D images at a time with fine frequency bands across the spectrum. Deriving 3D
72 information from these cameras is difficult as there is no overlap between the captured 1D
73 images (Hasheminasab et al., 2021). As for ranging sensors, Light Detection and Ranging
74 (LiDAR) can directly provide 3D points with high geometric accuracy. Nevertheless, the lack
75 of spectral/color information makes it difficult to derive semantic information for the acquired
76 scene. Due to the complementary characteristics of imaging and ranging sensors, the integration
77 of camera and LiDAR sensors can overcome their individual limitations, resulting in an
78 accurate and better description of the object space. This integration will enhance the process of
79 feature extraction, scene understanding, and visualization of derived products (Caltagirone et
80 al., 2019).

81 Meaningful integration of multi-temporal data/products from different modalities onboard
82 single or multiple systems is contingent on their positional quality. Accurate system calibration
83 – including the sensor’s interior orientation parameters (IOP) and mounting parameters relating
84 the sensors to the INS’ Inertial Measurement Unit (IMU) body frame – and trajectory
85 information are essential for ensuring the positional accuracy. Several studies have addressed
86 the problem of image/LiDAR integration by focusing on system calibration. Camera/LiDAR
87 calibration techniques estimate the system parameters by minimizing the discrepancy between
88 conjugate features extracted from both modalities through a Least Squares Adjustment (LSA)
89 procedure. Depending on the type of utilized features, calibration techniques can be categorized
90 into target-based and target-less approaches. For example, Zhang and Pless (2004) used a
91 planar checkerboard for establishing the mounting parameters relating camera and 2D LiDAR
92 units. Several studies extended this work for calibrating systems equipped with camera and 3D
93 LiDAR sensors (Mirzaei et al., 2012; Ravi et al., 2018; Verma et al., 2019). In addition to point-
94 to-plane geometric constraints, other feature correspondences such as line-to-plane (Zhou,
95 2014) and point-to-point (Beltran et al., 2022) have been also adopted using custom-built
96 targets. However, these approaches are time-consuming and might not be practical when
97 frequent system calibration is required. Early works dealing with in-situ calibration using
98 target-less strategies were based on manual identification of conjugate natural points and linear
99 features in indoor scenes (Moghadam et al., 2013; Scaramuzza et al., 2007). Several efforts
100 have been made toward developing fully-automated camera/LiDAR calibration frameworks.
101 The majority of these techniques – also referred to as motion-based approaches – use visual
102 odometry (Schneider et al., 2013) or SfM (Glira et al., 2016; Li et al., 2019; Zhou et al., 2021)
103 to establish conjugate features in image and LiDAR data. More specifically, these techniques
104 rely on deriving image-based point clouds and then matching those 3D points to LiDAR-
105 derived features. Trajectory information is usually refined in these approaches to achieve the
106 best alignment between camera and LiDAR data.

107 Despite that extensive body of work that has been conducted, there is still a need for a rigorous
108 integration approach that minimizes the discrepancy between camera and LiDAR data/products
109 caused by inaccurate system calibration parameters and/or trajectory artifacts. This study
110 proposes a semi-automated, tightly-coupled camera/LiDAR integration workflow for UAV
111 and wheeled remote sensing systems aided by a GNSS/INS unit. More specifically, the paper
112 presents a unified multi-sensor advanced triangulation (UMSAT), which can handle point,
113 linear, and areal features derived from imaging (e.g., frame cameras and push-broom scanners)
114 and ranging modalities aided by GNSS/INS position and orientation unit.

115 **2. Methodology**

116 The success of any multi-modal geospatial data integration activity is contingent on ensuring
117 the positional quality of such data (e.g., proper georeferencing of the used sensors together with
118 comprehensive modeling of the point positioning equations relating their measurements to the
119 respective ground coordinates). Before introducing the proposed UMSAT framework, the point
120 positioning models are first introduced. In general, establishing the point positioning equations
121 for either LiDAR or imaging systems proceeds in two steps. First, we need to define the laser

122 beam or imaging ray relative to the sensor coordinate system. This definition is based on the
 123 sensor measurements (i.e., laser range/pointing direction for a LiDAR and image coordinate
 124 measurements for a camera) together with the IOP of the used sensor (i.e., parameters
 125 describing the encoder mechanism for a LiDAR or principal point coordinates, principal
 126 distance, and distortion parameters for a camera). Second, the position and orientation of the
 127 laser beam or imaging ray relative to the mapping frame are established through the Exterior
 128 Orientation Parameters (EOP) that describe the position and orientation of the sensor relative
 129 to the mapping frame. For a GNSS/INS–assisted system, the EOP are derived using the post–
 130 processed GNSS/INS trajectory and mounting parameters relating these sensors to the
 131 corresponding IMU body frame.

132 The point positioning models for LiDAR and frame/line camera units are described in
 133 Equations (1)–(3), respectively. These models are also graphically explained in Figure 1. In
 134 Equation (1), $r_I^{lu(t)}$ denotes the position of the footprint of a laser beam, emitted at time t ,
 135 relative to the laser unit frame; $r_{b(t)}^m$ and $R_{b(t)}^m$ are the position and orientation of the IMU body
 136 frame relative to the mapping frame at time t ; r_{lu}^b and R_{lu}^b represent the lever arm and boresight
 137 rotation matrix relating the laser unit and IMU body frame coordinate systems. The derivation
 138 of $r_I^{lu(t)}$ is based on the range/pointing direction measurements of the LiDAR unit as well as
 139 its IOP. For the point positioning for frame and line imaging systems (Equations (2) and (3)),
 140 $r_i^{cf(t)}$ and $r_i^{cl(t)}$ represent the imaging rays for point i relative to the frame/line camera
 141 coordinate systems at time t . This term is derived from the image coordinates of point i (x_i and
 142 y_i) and camera IOP, including the principal point coordinates of used camera (x_p and y_p),
 143 principal distance (f), as well as distortions in the x and y coordinates for image point i ($dist_{x_i}$
 144 and $dist_{y_i}$). The main difference between frame and line cameras is that while both x , y
 145 components of image coordinates for the former have variable values depending on image point
 146 location, for the latter, the y coordinates are always constant – e.g., $y_i = 0$ for systems with the
 147 scan line vertically below the camera perspective center. r_{cf}^b/R_{cf}^b and r_{cl}^b/R_{cl}^b represent the lever
 148 arm and boresight rotation matrix relating the frame camera/line camera and IMU body frame
 149 coordinate systems. Different from LiDAR, image–based 3D reconstruction involves an
 150 unknown scale factor ($\lambda(i, cf, t)/\lambda(i, cl, t)$) for image point i captured by frame camera cf or
 151 line camera cl at time t , which needs to be estimated.

$$r_I^m = r_{b(t)}^m + R_{b(t)}^m r_{lu}^b + R_{b(t)}^m R_{lu}^b r_I^{lu(t)} \quad (1)$$

$$r_I^m = r_{b(t)}^m + R_{b(t)}^m r_{cf}^b + \lambda(i, cf, t) R_{b(t)}^m R_{cf}^b r_i^{cf(t)}, r_i^{cf(t)} = \begin{bmatrix} x_i - x_p - dist_{x_i} \\ y_i - y_p - dist_{y_i} \\ -f \end{bmatrix} \quad (2)$$

$$r_I^m = r_{b(t)}^m + R_{b(t)}^m r_{cl}^b + \lambda(i, cl, t) R_{b(t)}^m R_{cl}^b r_i^{cl(t)}, r_i^{cl(t)} = \begin{bmatrix} x_i - x_p - dist_{x_i} \\ 0 - y_p - dist_{y_i} \\ -f \end{bmatrix} \quad (3)$$

152

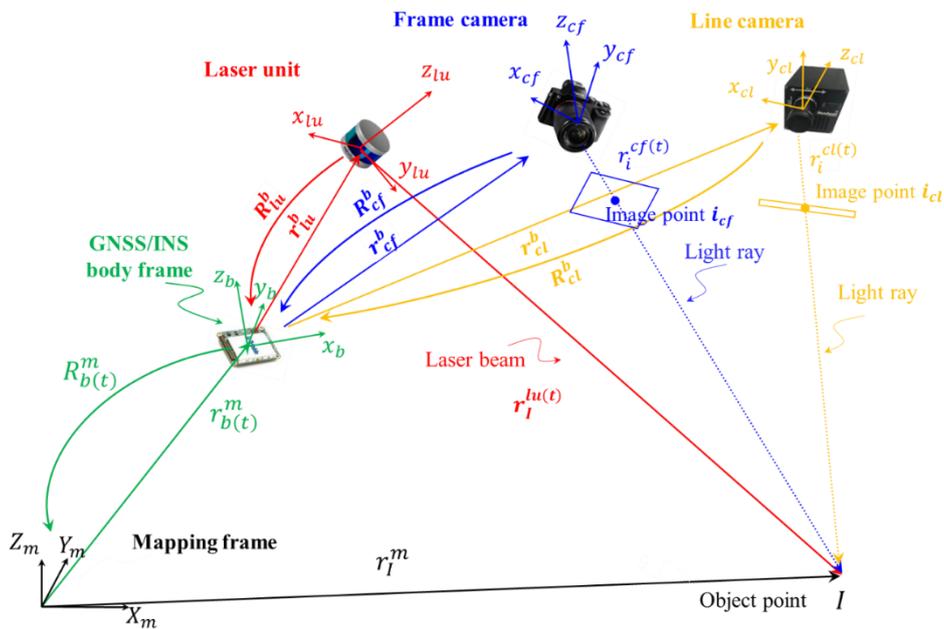


Figure 1. Schematic diagram of the point positioning principle for LiDAR and frame/line cameras units onboard a GNSS/INS–assisted MMS.

153 From the LiDAR/image–based point positioning equations, it is evident that accurate trajectory
 154 information and system calibration parameters (including sensor IOP and mounting parameters)
 155 are critical for producing properly georeferenced data from LiDAR and imaging systems.
 156 Therefore, to improve the positional quality of acquired data, a system–driven triangulation
 157 strategy is proposed. As illustrated in Figure 2, the triangulation aims at minimizing
 158 discrepancies among conjugate features (including point, linear, and areal features) captured by
 159 different sensor modalities from single/multiple systems through what will be denoted as
 160 universal multi–sensor advanced triangulation (UMSAT).

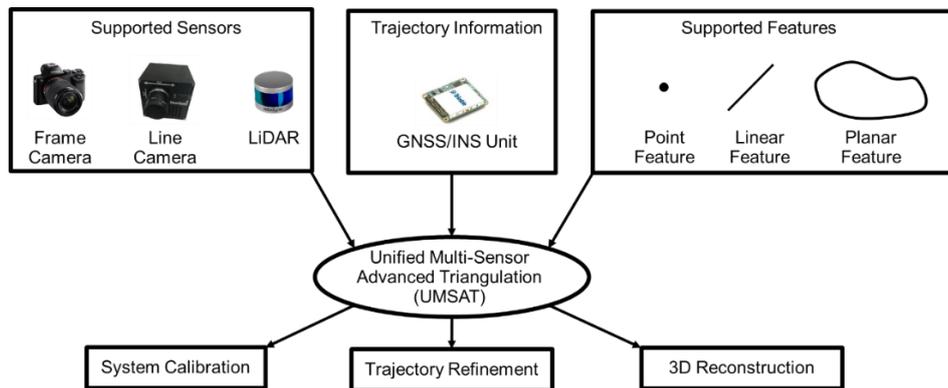


Figure 2. Schematic diagram of the functionality of the proposed UMSAT.

161 Point features are mainly adopted for cameras. The derivation of image–based object points can
 162 be conducted through SfM algorithms. For an object point I and its corresponding conjugate
 163 image points, the back–projection error is adopted as a cost function in UMSAT. Using the

164 frame camera as an example, the point positioning equation can be reformulated into Equation
 165 (4); to eliminate the unknown scale factor from this equation, the first and second rows are
 166 divided by the third one, and the image point coordinates are expressed as Equation (5). Based
 167 on this equation, the differences between observed image coordinates and predicted ones using
 168 estimated unknowns (i.e., back-projection errors) are minimized in the LSA process.

$$r_i^{cf(t)} = \frac{1}{\lambda(i, cf, t)} \left[R_b^{cf} R_m^{b(t)} (r_I^m - r_{b(t)}^m - R_{b(t)}^m r_{cf}^b) \right] = \frac{1}{\lambda(i, cf, t)} \begin{bmatrix} N_x \\ N_y \\ D \end{bmatrix} \quad (4)$$

$$x_i = -c \frac{N_x}{D} + x_p + dist_{x_i}$$

$$y_i = -c \frac{N_y}{D} + y_p + dist_{y_i} \quad (5)$$

169 In terms of linear features, they can be extracted from images and LiDAR data. For imagery,
 170 the Line Segment Detector (LSD) (Grompone Von Gioi et al., 2010) can be used to
 171 automatically derive image linear features. For LiDAR data, linear features could be derived
 172 through a segmentation process followed by Principal Component Analysis (PCA) analysis. In
 173 UMSAT, an object space linear feature is represented by two endpoints P_A and P_B . Two
 174 optimization target functions are implemented for minimizing the discrepancies between
 175 conjugate image/LiDAR lines. The involved quantities in the two target functions are
 176 schematically illustrated in Figure 3. The LiDAR optimization target function minimizes the
 177 normal distance between the mapping coordinates of a LiDAR point I and the linear feature it
 178 belongs to (defined by the two endpoints P_A and P_B). This constraint is mathematically
 179 described in Equation (6), where $\|x\|$ represents the L_2 -norm of the vector x , $r_{P_A}^m$ and $r_{P_B}^m$
 180 are the object coordinates of the two endpoints, and r_I^m is the coordinates of LiDAR point I . The
 181 image optimization target function forces the vector from the perspective center (PC) to an
 182 image point i along the linear feature (r_{PC-i}^m) to lie on the plane defined by the PC and endpoints
 183 of the object line (i.e., the plane defined by vectors $r_{PC-P_A}^m$ and $r_{PC-P_B}^m$, as shown in Figure 3).
 184 This constraint is mathematically presented by the triple product in Equation (7), where $r_{PC-P_A}^m$
 185 is defined by $r_{P_A}^m - r_{c(t)}^m$ with $r_{c(t)}^m$ representing the camera position relative to the mapping
 186 frame at time t ; and r_{PC-i}^m is the vector from the camera perspective center to an intermediate
 187 image point i along the line in the mapping frame as represented as $R_{b(t)}^m R_c^b r_i^{c(t)}$. To analyze
 188 the residual of such constraint following the LSA, the angle α between the vector r_{PC-i}^m and the
 189 plane defined by the PC and object line endpoints P_A/P_B (as shown in Figure 3) is evaluated.

190 Areal features are only used for LiDAR sensors as they cannot provide redundant information
 191 for imagery. Areal features can be automatically extracted from LiDAR data through various
 192 approaches. In UMSAT, areal features are modeled as planes. The respective target function
 193 minimizes the normal distance between the LiDAR point I to the areal feature it belongs to, as
 194 mathematically described in Equation (8). In this equation, A, B, C, D are the plane parameters
 195 and (X_I, Y_I, Z_I) are the coordinates of LiDAR point I in the mapping frame.

$$\frac{\|(r_{P_B}^m - r_{P_A}^m) \times (r_{P_B}^m - r_I^m)\|}{\|r_{P_B}^m - r_{P_A}^m\|} = 0 \quad (6)$$

$$(r_{P.C.-A}^m \times r_{P.C.-B}^m) \cdot r_{P.C.-i}^m = \left([r_{P_A}^m - [r_{b(t)}^m + R_{b(t)}^m r_c^b]] \times [r_{P_B}^m - [r_{b(t)}^m + R_{b(t)}^m r_c^b]] \right) \cdot R_{b(t)}^m R_c^b r_i^{c(t)} = 0 \quad (7)$$

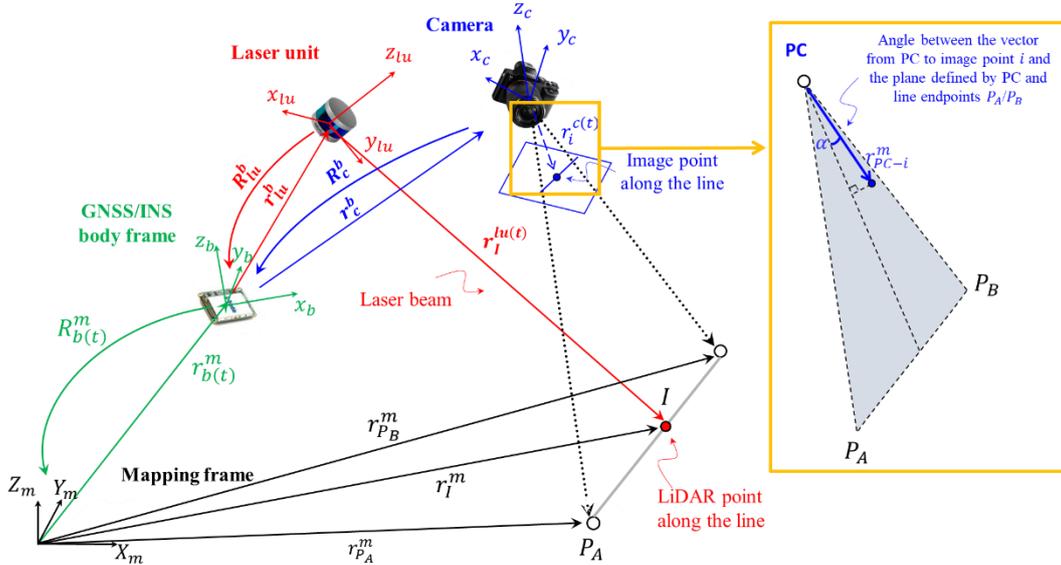


Figure 3. Schematic illustration of image/LiDAR points along the linear feature for the SMART system (points I and i represent the points along the linear feature observed by LiDAR and camera, respectively).

$$AX_I + BY_I + CZ_I + D = 0 \quad (8)$$

196 The involved parameters in the above optimization functions include the respective sensor's
 197 IOP and mounting parameters, trajectory information at the timestamp of the observation, and
 198 parameters for the respective object point, linear, or areal feature. In this study, trajectory
 199 information is refined by estimating corrections $(\delta r_{b(t)}^m, \delta R_{b(t)}^m)$ to the original
 200 position/orientation parameters derived from the post-processed GNSS/INS observations.
 201 Solving for the trajectory corrections at the timestamp for every camera/LiDAR observation is
 202 not recommended as it would cause over-parametrization in the LSA. Since we are dealing
 203 with a platform that has a relatively smooth trajectory with moderate dynamics, the original
 204 high-frequency (e.g., 100–200 Hz) trajectory is down-sampled (i.e., using a down-sampling
 205 time interval ΔT). The down-sampled trajectory points are henceforth denoted as trajectory
 206 reference points, as shown in Figure 4. The corrections to the trajectory parameters at a specific
 207 observation timestamp are then modeled as a p^{th} -order polynomial function of the unknown
 208 corrections at its n neighboring trajectory reference points. Symbolically, this polynomial
 209 modeling is expressed in Equation (9), where it can be seen that for a generic timestamp, T_0 , its
 210 trajectory corrections (denoted generically as $\delta \theta_{b(T_0)}^m$) are a function of the timestamps and
 211 trajectory corrections of its n neighboring trajectory reference points. The down-sampling time

212 interval, polynomial order, and number of neighboring trajectory reference points are chosen
 213 based on the nature of platform dynamics.

$$\delta\theta_{b(T_0)}^m = f(T_0, T_i, T_{i+1}, \dots, T_{i+n-1}, \delta\theta_{b(T_i)}^m, \delta\theta_{b(T_{i+1})}^m, \dots, \delta\theta_{b(T_{i+n-1})}^m) \quad (9)$$

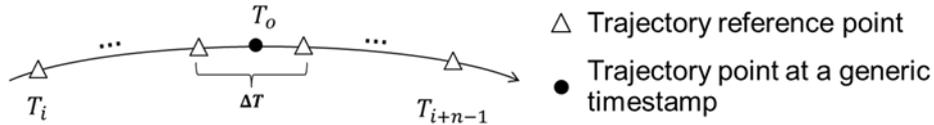


Figure 4. Down-sampled trajectory reference points (with down-sampling time interval ΔT) used for trajectory enhancement: T_i to T_{i+n-1} denote the n neighboring trajectory reference points for a generic timestamp T_0 .

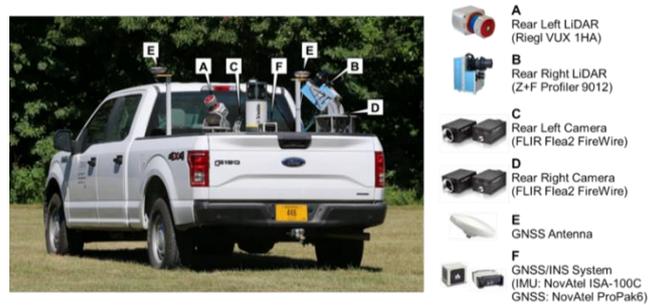
214 Other than the above target functions, other constraints are also included in the UMSAT. To
 215 guarantee the smoothness/continuity of the refined trajectory, correction differences for
 216 successive trajectory reference points are minimized. For an image-based object point P that
 217 belongs to an areal LiDAR feature, the normal distance between P and the corresponding plane
 218 can be also minimized in the LSA. Similarly, for a linear-areal feature correspondence, the
 219 endpoints to plane distances can be minimized. These correspondences are useful to integrate
 220 various camera and LiDAR features. Finally, UMSAT supports the incorporation of prior
 221 information for the unknowns including system calibration parameters, trajectory information,
 222 and/or feature parameters. This is conducted through the incorporation of pseudo observations.

223 3. Utilized Mobile Mapping Systems and Datasets Description

224 3.1. Wheeled and UAV Mobile Mapping Systems

225 This study involves two in-house developed wheeled MMS – Purdue wheel-based mobile
 226 mapping system–Ultra High Accuracy (PWMMS–UHA) and Purdue wheel-based mobile
 227 mapping system–High Accuracy (PWMMS–HA). The PWMMS–UHA, as displayed in Figure
 228 5a, is equipped with two single-beam LiDAR scanners: Riegl VUX 1HA and Z+F Profiler
 229 9012. These scanners deliver a 360° horizontal field of view (FOV). Each scanner can deliver
 230 upto 1,000,000 points per second. Two rear-facing FLIR Flea2 FireWire cameras are installed
 231 on the PWMMS–UHA. Both cameras have a maximum image resolution of 5.0 MP and are
 232 synchronized to capture images at a rate of 1 frame every 0.75 s. All sensors are directly
 233 georeferenced by a NovAtel ProPak6 and ISA–100C GNSS/INS unit. The PWMMS–HA, as
 234 shown in Figure 5b, includes four multi-beam LiDAR scanners: three Velodyne HDL–32Es
 235 and one Velodyne VLP–16 Hi–Res. The HDL–32E consists of 32 radially oriented laser
 236 rangefinders aligned vertically from -30.67° to $+10.67^\circ$. The VLP–16 Hi–Res has 16 radially
 237 oriented laser rangefinders with a 20° vertical FOV. The point capture rates for HDL–32E and
 238 VLP–16 Hi–Res are 700,000 and 300,000 points per second, respectively. Three FLIR
 239 Grasshopper3 9.1MP GigE cameras are also mounted on the PWMMS–HA: two forward-
 240 facing and one rear-facing. The cameras are synchronized to capture one 1 frame per second
 241 per camera. The PWMMS–HA sensors are directly georeferenced by an Applanix POS LV 220

242 GNSS/INS unit. In addition to wheeled MMS, an off-the-shelf UAV system, a DJI M300
 243 equipped with the Zenmuse L1 LiDAR sensor is used (Figure 5c). The Zenmuse L1 integrates
 244 a Livox LiDAR module, a camera, and an IMU on a 3-axis stabilized gimbal. The LiDAR
 245 horizontal and vertical FOVs are 70.4° and 4.5°, respectively. The point capture rate is 700,000
 246 points per second. The UAV camera has a 1-inch CMOS with a 24 mm focal length and a
 247 maximum image resolution of 20.7 MP. The IMU unit has a measurement rate of 200 Hz. After
 248 post-processing, a position accuracy of ± 1 to ± 1.5 cm and attitude accuracy of $\pm 0.025^\circ$ and
 249 $\pm 0.15^\circ$ for pitch/roll and heading, respectively, can be achieved.



(a)



(b)



(c)

Figure 5. Illustrations of (a) Purdue wheel-based MMS—ultra high accuracy system (PWMMS-UHA), (b) Purdue wheel-based MMS—high accuracy system (PWMMS-HA), and (c) DJI M300 UAV equipped with the Zenmuse L1 (adapted from DJI website).

250 3.2 Study Site and Datasets Description

251 In this study, the image/LiDAR data were acquired
252 along a 0.5 mile segment of the I-65 highway close to
253 Lebanon, IN, United States, as shown in Figure 6. The
254 site is rich with point, linear, and areal features, which
255 could be identified in the imagery and LiDAR data. A
256 total of three datasets are used.



Figure 6. Location of study site adopted in this research.

257 Table 1 provides a summary of the relevant information
258 for the datasets acquired from the PWMMS-UHA,
259 PWMMS-HA, and DJI systems.

260 Table 1. Specifications of acquired datasets for the wheeled and UAV systems.

Platform	Data Acquisition Date	Driving/flight Speed	Number of Collected LiDAR Points (in millions)	Number of Captured Images
PWMMS-UHA	2022.07.10	~50 mph	~23	52
PWMMS-HA	2022.12.02	~50 mph	~63	158
DJI UAV*	2022.08.02	~8.5 mph	~85	88

261 * Above ground flying height is 70 meters.

262 **4. Experimental Results**

263 In this study, camera/LiDAR data collected by the three systems are integrated through the
264 proposed UMSAT framework. LiDAR data acquired by the DJI system is used as a reference
265 due to the high positional accuracy provided through uninterrupted GNSS-signal. For the
266 captured UAV images, we have only access to approximate geotagging information. While
267 camera IOP were derived through an SfM process, the EOP are estimated through UMSAT.
268 For the wheeled MMS, rigorous system calibration has been conducted. Although onboard
269 GNSS/INS units provide trajectory with reasonable accuracy, misalignments can be observed
270 due to GNSS-signal interruptions for the wheeled MMS units. Therefore, system calibration
271 parameters (IOP and mounting parameters for the camera and LiDAR units) of PWMMS-UHA
272 and PWMMS-HA are fixed in the LSA while refining their trajectory through UMSAT.

273 For the triangulation features, lane markings are modeled as linear features, as shown in Figure
274 7. Specifically, skip-lines are modeled as individual linear features. Edge lines, on the other
275 hand, are divided into short straight line segments. A geometry-based approach is adopted to
276 extract lane markings from the UAV and wheeled LiDAR data (Cheng et al., 2020). Then, lane
277 markings derived from PWMMS-UHA, PWMMS-HA, and DJI LiDAR are matched according
278 to their spatial proximity. Corresponding image lines are established through back-projection
279 of LiDAR linear features onto the imagery. Since lane markings are almost parallel, they only
280 provide control in the vertical and across driving directions. Therefore, four poles (colored in
281 red in Figure 7) are manually extracted from the LiDAR data to provide control along the

282 driving direction. These poles are modeled as linear features as well. For the UAV camera
283 imagery, other than the manually established linear features, a SfM is conducted to derive image
284 tie points with the corresponding object points depicted in Figure 7. Moreover, intermediate
285 points along the lane markings and poles have been also established in the imagery.

286 In this study, the endpoints of line segments (including lane markings and poles) from the DJI
287 UAV LiDAR are used as control information. The PWMMS–UHA (with two LiDAR units and
288 two cameras), PWMMS–HA (with four LiDAR units and three cameras), and UAV camera
289 datasets are included by considering both point and linear features. For the two wheeled
290 systems, the trajectory is modeled by 1HZ reference points with a 2nd order polynomial. For the
291 UAV images, we only solve for position/orientation corrections at their locations (i.e., a zero-
292 order polynomial is used – i.e., a reference point is defined for each image).

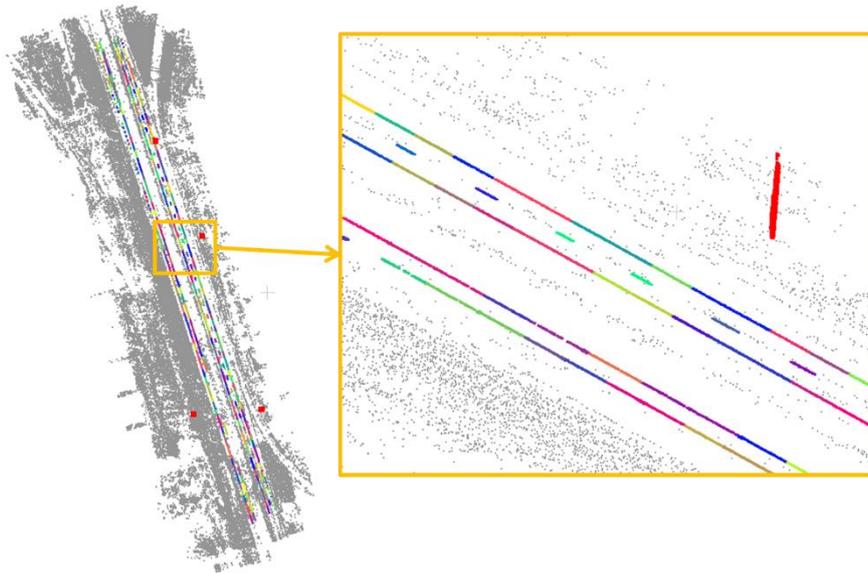


Figure 7. Established lane markings (randomly colored by the feature ID) and four poles (colored in red) from the DJI LiDAR data, as well as the image-based object points (colored in grey) derived from SfM processing of the DJI imagery.

293 The performance of UMSAT is first evaluated through qualitative analysis. Specifically,
294 profiles perpendicular to the driving direction are extracted from the DJI, PWMMS–UHA, and
295 PWMMS–HA LiDAR data as well as the image-based point cloud from the DJI camera (Figure
296 8). We can observe that the misalignment in the across driving and Z directions is minimized
297 after the UMSAT process. To evaluate the alignment along the driving direction, one of the
298 light poles is extracted from the profiles and also shown in Figure 8, where we can see good
299 alignment along the driving direction (refer to the close-up views in Figure 8).

300 Having examined the 3D alignment between the LiDAR and image-based point clouds, the
301 accuracy of the camera geo-tagging for all systems is evaluated through backward projection,
302 as shown in Figure 9. A point on one of the light poles was selected in the DJI LiDAR data and

303 back-projected onto the images from DJI, PWMMS-HA, and PWMMS-UHA systems. As
 304 shown in Figure 9b, the 2D misalignment is minimized following the UMSAT optimization.

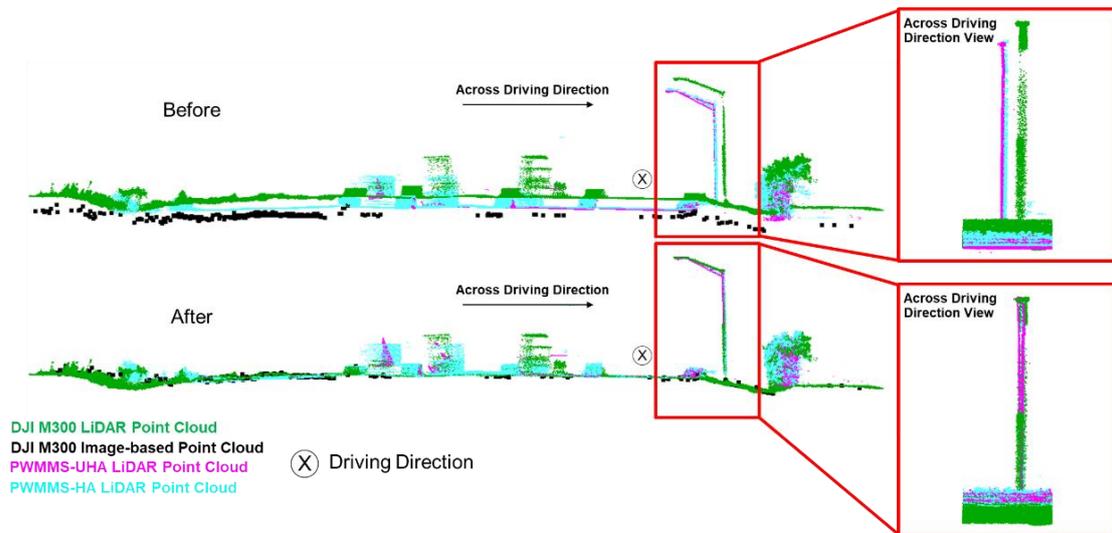


Figure 8. Illustration of extracted profiles/features from the LiDAR data as well as image-based point cloud before/after UMSAT optimization showing improved alignment in the across/along driving and vertical directions.

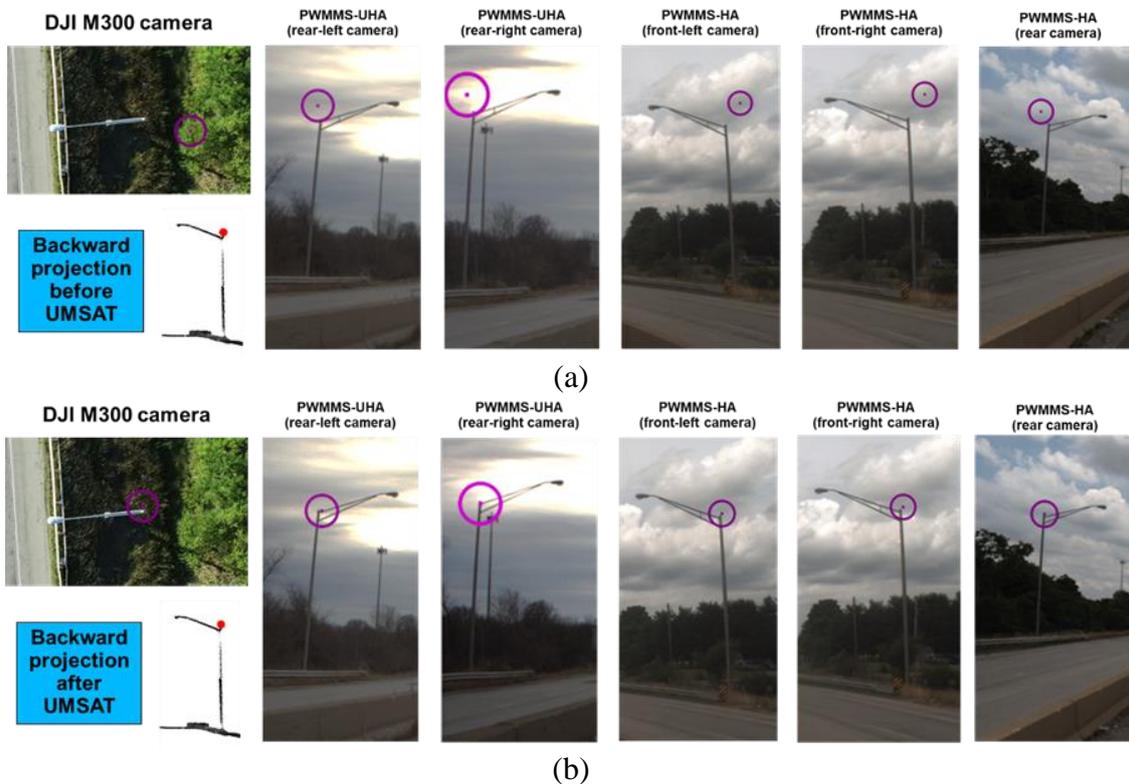


Figure 9. Illustration of image back-projection accuracy (a) before and (b) after applying UMSAT.

305 The quantitative evaluation of UMSAT is analyzed through the Root Mean Square (RMS)
 306 values of residuals for the camera/LiDAR constraints, including (i) normal distance from
 307 LiDAR points to the respective object-space linear features, (ii) α angle between the imaging
 308 ray for an intermediate point and the respective plane through the linear feature, and (iii) back-
 309 projection error for image tie points. Table 2 lists the above metrics before and after the UMSAT
 310 optimization for the different systems. It can be seen from this table that the before optimization
 311 misalignment, in the range of 1.2 – 1.3 m for the LiDAR linear features, is reduced to roughly
 312 7 cm. As for the image linear features, the RMS of the α angle is reduced to almost 0.2°. As for
 313 the image tie points, a 1.3 pixel back-projection error is achieved for the DJI camera data.

314 Table 2. Quantitative evaluation of the pre/post-UMSAT optimization for the three datasets.

	RMS of normal distances for the LiDAR linear features (m)		RMS of α angles for the image linear features (degree)		RMS of back-projection errors for the image tie points (pixel)	
	Before	After	Before	After	Before	After
Overall	1.262	0.069	3.250	0.191	6.872	1.319
PWMMS-UHA	1.307	0.065	3.716	0.269	N/A	N/A
PWMMS-HA	1.222	0.073	5.250	0.213	N/A	N/A
DJI Camera	N/A	N/A	0.915	0.140	6.872	1.319

315

316 5. Conclusions and Recommendations for Future Work

317 This study presented a unified multi-sensor advanced triangulation (UMSAT), which can
 318 handle point, linear, and areal features derived from imaging and ranging remote sensing
 319 systems aided by a GNSS/INS position and orientation unit. Camera/LiDAR data collected by
 320 two wheeled and one UAV MMS over a highway study site are used to evaluate the
 321 performance of the proposed strategy. Experimental results indicate that through UMSAT,
 322 camera and LiDAR data from these systems are well-aligned (i.e., indicating that multi-
 323 temporal, multi-sensor, and multi-platform geospatial data are ready for subsequent integration
 324 activities). The current limitation of the proposed strategy is the time-consuming, manual
 325 measurements of image linear features. Therefore, automated feature extraction and matching
 326 procedures from camera and LiDAR data will be explored. Moreover, the feasibility of using
 327 UMSAT in GNSS-challenging, and potentially GNSS-denied, environments will be
 328 investigated.

329

330 REFERENCES

- 331 Beltran, J., Guindel, C., De La Escalera, A., & Garcia, F. (2022). Automatic Extrinsic
 332 Calibration Method for LiDAR and Camera Sensor Setups. *IEEE Transactions on*
 333 *Intelligent Transportation Systems*, 23(10). <https://doi.org/10.1109/TITS.2022.3155228>
 334 Caltagirone, L., Bellone, M., Svensson, L., & Wahde, M. (2019). LiDAR-camera fusion for
 335 road detection using fully convolutional neural networks. *Robotics and Autonomous*
 336 *Systems*, 111, 125–131. <https://doi.org/10.1016/j.robot.2018.11.002>
 337 Cheng, Y. T., Patel, A., Wen, C., Bullock, D., & Habib, A. (2020). Intensity thresholding and
 338 deep learning based lane marking extraction and lanewidth estimation from mobile light

- 339 detection and ranging (LiDAR) point clouds. *Remote Sensing*, 12(9).
340 <https://doi.org/10.3390/RS12091379>
- 341 Glira, P., Pfeifer, N., & Mandlbürger, G. (2016). Rigorous Strip adjustment of UAV-based
342 laserscanning data including time-dependent correction of trajectory errors.
343 *Photogrammetric Engineering & Remote Sensing*, 82(12), 945–954.
344 <https://doi.org/10.14358/PERS.82.12.945>
- 345 Grompone Von Gioi, R., Jakubowicz, J., Morel, J. M., & Randall, G. (2010). LSD: A fast line
346 segment detector with a false detection control. *IEEE Transactions on Pattern Analysis
347 and Machine Intelligence*, 32(4). <https://doi.org/10.1109/TPAMI.2008.300>
- 348 Guan, H., Li, J., Yu, Y., Wang, C., Chapman, M., & Yang, B. (2014). Using mobile laser
349 scanning data for automated extraction of road markings. *ISPRS Journal of
350 Photogrammetry and Remote Sensing*, 87. <https://doi.org/10.1016/j.isprsjprs.2013.11.005>
- 351 Hasheminasab, S. M., Zhou, T., Laforest, L., & Habib, A. (2021). Multiscale Image Matching
352 for Automated Calibration of UAV-Based Frame and Line Camera Systems. *IEEE
353 Journal of Selected Topics in Applied Earth Observations and Remote Sensing*, 14.
354 <https://doi.org/10.1109/JSTARS.2021.3062573>
- 355 Li, J., Yang, B., Chen, C., & Habib, A. (2019). NRLI-UAV: Non-rigid registration of
356 sequential raw laser scans and images for low-cost UAV LiDAR point cloud quality
357 improvement. *ISPRS Journal of Photogrammetry and Remote Sensing*, 158.
358 <https://doi.org/10.1016/j.isprsjprs.2019.10.009>
- 359 Mirzaei, F. M., Kottas, D. G., & Roumeliotis, S. I. (2012). 3D LiDAR-camera intrinsic and
360 extrinsic calibration: Identifiability and analytical least-squares-based initialization. *The
361 International Journal of Robotics Research*, 31(4), 452–467.
362 <https://doi.org/10.1177/0278364911435689>
- 363 Moghadam, P., Bosse, M., & Zlot, R. (2013). Line-based extrinsic calibration of range and
364 image sensors. *Proceedings – IEEE International Conference on Robotics and
365 Automation*. <https://doi.org/10.1109/ICRA.2013.6631095>
- 366 Nex, F., & Remondino, F. (2014). UAV for 3D mapping applications: A review. In *Applied
367 Geomatics* (Vol. 6, Issue 1). <https://doi.org/10.1007/s12518-013-0120-x>
- 368 Ravi, R., Lin, Y.-J., Elbahnasawy, M., Shamseldin, T., & Habib, A. (2018). Simultaneous
369 system calibration of a multi-LiDAR multicamera mobile mapping platform. *IEEE
370 Journal of Selected Topics in Applied Earth Observations and Remote Sensing*, 11(5),
371 1694–1714. <https://doi.org/10.1109/JSTARS.2018.2812796>
- 372 Scaramuzza, D., Harati, A., & Siegwart, R. (2007). Extrinsic self calibration of a camera and a
373 3D laser range finder from natural scenes. *IEEE International Conference on Intelligent
374 Robots and Systems*. <https://doi.org/10.1109/IROS.2007.4399276>
- 375 Schneider, S., Luettel, T., & Wuensche, H. J. (2013). Odometry-based online extrinsic sensor
376 calibration. *IEEE International Conference on Intelligent Robots and Systems*.
377 <https://doi.org/10.1109/IROS.2013.6696515>
- 378 Verma, S., Berrio, J. S., Worrall, S., & Nebot, E. (2019). Automatic extrinsic calibration
379 between a camera and a 3D Lidar using 3D point and plane correspondences. *2019 IEEE
380 Intelligent Transportation Systems Conference, ITSC 2019*.
381 <https://doi.org/10.1109/ITSC.2019.8917108>

- 382 Westoby, M. J., Brasington, J., Glasser, N. F., Hambrey, M. J., & Reynolds, J. M. (2012).
383 “Structure–from–Motion” photogrammetry: A low–cost, effective tool for geoscience
384 applications. *Geomorphology*, 179. <https://doi.org/10.1016/j.geomorph.2012.08.021>
385 Zhang, Q., & Pless, R. (2004). Extrinsic calibration of a camera and laser range finder (improves
386 camera calibration). *IROS*, 3, 2301–2306. <https://doi.org/10.1109/IROS.2004.1389752>
387 Zhou, L. (2014). A new minimal solution for the extrinsic calibration of a 2D LIDAR and a
388 camera using three plane–line correspondences. *IEEE Sensors Journal*, 14(2).
389 <https://doi.org/10.1109/JSEN.2013.2284789>
390 Zhou, T., Hasheminasab, S. M., & Habib, A. (2021). Tightly–coupled camera/LiDAR
391 integration for point cloud generation from GNSS/INS–assisted UAV mapping systems.
392 *ISPRS Journal of Photogrammetry and Remote Sensing*, 180.
393 <https://doi.org/10.1016/j.isprsjprs.2021.08.020>

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